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Bilateral Teleoperation of Multiple UAVs with Decentralized Bearing-only Formation Control

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***Abstract*— We present a decentralized system for the bilateral teleoperation of groups of UAVs which only relies on relative bearing measurements, i.e., without the need of distance information or global localization. The properties of a *3D bearing-formation* are analyzed, and a minimal set of bearings needed for its definition is provided. We also design a novel decentralized formation control almost globally convergent and able to maintain bounded and non-vanishing inter-distances among the agents despite the absence of direct distance measurements. Furthermore, we develop a multi-master/multi-slave teleoperation setup in order to control the overall behavior of the group and to convey to the human operator suitable force cues, while ensuring stability in presence of delays and packet losses over the master-slave communication channel. The theoretical framework is validated by means of extensive human/hardware in-the-loop simulations using two force-feedback devices and a group of quadrotors.**

I. INTRODUCTION

Teleoperation of a group of mobile robots in a bilateral way (i.e., providing a force feedback to the human operator) is an emerging topic which combines autonomous multi-robot system research with the studies on human-robot interaction. This topic received a rapidly-increasing attention in recent years, starting from [1], [2], [3] up to [4] and [5].

The use of a system with multiple deployable agents instead of a bulky robot presents undoubtable advantages, since a decentralized solution increases the robustness and flexibility of the robotic system. These advantages constitute the main motivation between the high number of recent publications in this field, see [6], [7], [8] just to cite a few. On the other hand, in real scenarios, the complete autonomy of multiagent systems is still far from being a reality because of the complexity and unpredictability of the environment as, e.g., in search and rescue missions. Therefore, in many practical cases, the use of a semi-autonomous group of robots partially guided by a human operator still represents the only viable solution. As an alternative to unilateral teleoperation, the use of suitable sensorial feedback has also been proved to improve the (tele-)presence of the human operator, in particular by exploiting the haptic (force-feedback) sense [9]. It is then interesting to study the possibility of establishing a bilateral teleoperation channel interfacing a human operator with a remote group of agents possessing some local autonomy, but still bound to follow the high-level human motion commands. We focus our attention to the particular case of a swarm of Unmanned Aerial Vehicles (UAVs) because of their adaptability and potential pervasiveness to many different scenarios. Nevertheless our results may be seamlessly extended to ground, marine and submarine robots.

In this paper, for the sake of autonomy, we designed our system to be independent from the knowledge of the absolute position in space of the UAVs. In addition, we aimed for a solution using standard, light-weight, low-energy, and cheap sensors. A camera, for example, well fits these requirements, especially when compared to other active sensing devices like laser/structured-light range-sensors. We also decided to not use stereo-cameras since the typical aerial distances would require a too large baseline in order to extract the depth. In addition we avoid any depth estimation and we delegate to the human operator the role of regulating the expansion/contraction rate of the swarm. Therefore we assume that the inter-sensing system between UAVs (e.g., cameras) provides only the relative bearings (i.e., the pointing directions in the sensor frame).

In our teleoperation scenario the slave side is made of a group of UAVs able to only measure the reciprocal (relative) bearings and to control their linear and angular speed. At the beginning of the task the human user selects a set of desired relative bearings in order to restrict the motion of the group to a shape which is scale-, placement-, and rotational-invariant (a *bearing-formation*). The desired relative bearings are typically chosen in order to optimize some useful criteria, e.g., guarantee inter-UAV visibility or environmental coverage. The swarm autonomously achieves the desired bearing-formation without collapsing or indefinitely expanding (despite the fact that the inter-distances are not measured). The operator controls the translational velocity of the formation and its dilation rate, by manipulating the position of two haptic devices, and receives a force feedback proportional to the velocity and expansion-rate tracking error of the UAVs.

The main contributions of this paper follows: we describe and analyze rigorously the concept of bearing-formation. We find the minimal set of bearings needed to uniquely define a bearing-formation, with linear cardinality in the number of robots. We design a bearing-formation control for aerial agents which uses only the relative bearings, converges almost globally, stabilizes the inter-distances to a finite value, and does not need any persistent excitation to accomplish the task. We propose a new system for the bilateral teleoperation of a group of UAVs using only vision sensors, instead of distance ones, and we use a multi-master/multi-slave approach where one master is used to control the translation and the second one to regulate the expansion rate. We ensure stability

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of the system in presence of delays and packet losses on the master-slave communication channel. Finally, we validate the theory by means of extensive human/hardware-in-the-loop simulations with a group of quadrotors and two force-feedback devices (as shown in the accompanying video).

The paper is organized as follows: in Sec. I-A we present a literature review. In Sec. II we model the tele-operated multi-robot system. In Sec. III and IV we analyze the bearing-formation and design the controllers for the slave side. Section V illustrates the master-side controller and Sec. VI presents the human/hardware-in-the-loop simulations. Finally, Sec. VII concludes the paper and the Appendix provides a brush-up on polar coordinates and all the proofs.

A. Related Work

A passivity-based approach for the bilateral teleoperation (*BT*) of a group of holonomic/non-holonomic ground robots is presented in [1], [2]. In [3] UAV-BT is performed coupling the *position* of a master device to the *position* of the centroid of the formation. On the contrary, [5] presents a UAV-BT scheme where the *position* of the master device controls the *velocity* of the centroid. In [4] a decentralized UAV-BT control strategy based on a leader-follower approach is proposed which allows the formation to split and rejoin in a passive way. All the aforementioned approaches need, directly or indirectly, to measure the inter-distance between the robots in order to ensure the slave-side formation control.

In the literature on autonomous formation control the use of bearing measurements has been mostly considered for groups of *non-holonomic* ground robots, with a special regard on leader-follower configurations. In [10] a leader-follower control is proposed based on input-output feedback linearization. In [11] distance is achieved using an Extended Kalman Filter and a neural network. In [12] a leader-follower approach based on feedback linearization is proposed. In [13] parallel and circular constant speed formation are obtained using the bearing angle, optical flow and time to collision. The control objective of all the aforementioned approaches includes the regulation of the inter-distances (estimated from the bearings) to a specified value Therefore, they require the whole formation to keep moving in order to achieve the desired bearings and inter-distances. This persistent-excitation behavior is not needed in our approach.

Finally, a consensus-based approach [14] is also not adequate for us since it would require knowledge of the inter-distances between the UAVs.

II. THE SLAVE SIDE

The slave-side of the proposed teleoperation system is composed of N UAVs, modeled as rigid bodies in space, therefore the configuration of each one is described by a point in $SE(3)$. Denote with $\boldsymbol{q}_i^\star = (\boldsymbol{p}_i^\star, \theta_i^\star) \in \mathbb{R}^3 \times S^1$ the centroid and the yaw angle of the UAV respectively. We assume that the i-th UAV is able to track any smooth reference trajectory $(\boldsymbol{p}_i(t), \theta_i(t))$ with $(\boldsymbol{p}_i^\star, \theta_i^\star)$. A sufficient condition for the previous assumption is that the position of the centroid and the yaw angles are flat outputs [15], or, equivalently, that the UAV is dynamically feedback linearizable. It is well known that both helicopters and quadrotors meet this property [16], [17]. A description of the particular trajectory controller used to track the reference trajectory is outside the scope of this paper, as an example, we refer the reader to [18], [19] where related controllers for quadrotors are proposed. As a matter of fact, the configuration of the real UAV will not exactly track the reference trajectory due to, e.g., inertia, actuator limits, sensors noise, and air drag. We assume that the UAV is endowed with a trajectory tracking controller allowing to follow the velocity commands with a good performance, by keeping tracking errors small enough. This is a common assumption in literature (e.g., see [7], [8]).

In order to produce online an effective reference trajectory $\boldsymbol{p}_i(t), \theta_i(t)$ for the i-th UAV, we use a *trajectory planner*, i.e, a kinematic system (henceforth called *agent*) whose state is the pair $\boldsymbol{q}_i = (\boldsymbol{p}_i,\ \theta_i) \in \mathbb{R}^3 \times S^1$ and inputs are the linear velocity $\boldsymbol{u}_i$ and the yaw-rate ω_i:

$$\begin{pmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\theta}_i \end{pmatrix} = \begin{pmatrix} R_i & \mathbf{0} \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_i \\ \omega_i \end{pmatrix}, \qquad (1)$$

where $\mathbf{0} = (0\,0\,0)^T$, and matrix R_i represents a rotation of angle θ_i around the z axis. We denote the inverse/transpose of R_i with ${}^iR = R_i^T$, and the rotation between two body frames with ${}^iR_j = {}^iRR_j$. The reduced configuration space and the related kinematic model (1) stem from practical foundation. In fact in the real world, even if no global positioning system is given, measurement of the direction of gravity is quite reliable and available everywhere by means of an accelerometer, from which the UAV can obtain roll and pitch angles. On the contrary, measurement of an additional inertial direction (such as the the north with a compass) is often unreliable or not available at all (e.g., indoor, or in presence of exogenous magnetic fields).

In our context a *bearing* is a unit vector in $\mathbb{R}^3$, i.e., a point in $\mathbb{S}^2$, the unit sphere. The i-th agent measures the *relative bearing*, in its body frame, w.r.t. the j-th agent:

$${}^i\boldsymbol{\beta}_{ij} = {}^iR\,\boldsymbol{p}_{ij}/\delta_{ij}, \qquad (2)$$

where $\boldsymbol{p}_{ij} = \boldsymbol{p}_j - \boldsymbol{p}_i$, and $\delta_{ij} = \|\boldsymbol{p}_j - \boldsymbol{p}_i\|$ is the inter-distance between agent i and j. The i-th agent is not able to measure either the inter-distance δ_{ij} w.r.t. any $j \neq i$, or its absolute configuration $\boldsymbol{q}_i$, i.e., its world frame configuration. Relative bearings can be measured directly from the image of a monocular camera and the knowledge of roll and pitch angles. On the contrary, inter-distance is not immediately available from the image. A possibility to recover this information would require the exact knowledge of the real agent dimensions to be compared with its apparent size. Unfortunately, the computation of the apparent size is not feasible with normal cameras in the range of the typical aerial distances. Lastly notice that the model (1–2) may describe both a UAV with a camera mounted on a pan unit, as well as UAV which can turn on the z axis independently from the direction of motion, e.g., the quadrotors used in Sec. VI.

The first objective of the slave-side is to autonomously keep the desired *bearing-formation*, i.e., to stay in the equivalence class of formations specified giving a set of desired relative bearings to every agent. These kinds of formations are analyzed in Sec. III. The second objective of the slave-side is to allow the operator to steer the overall motion of the formation while keeping the desired bearing-formation. In Sec. III, Prop. 3, we will show that the motions not affecting all the bearings (i.e., tangent to a bearing-formation) are a linear combination of 3 types: (1) synchronized translation, (2) dilation, and (3) rotation around a line parallel to the z axis of the world frame. We will also show that inter-distances are necessarily needed to perform a synchronized rotation, although knowledge of the bearings only is sufficient to realize the other two synchronized maneuvers, namely translation and dilation. For this reason we will allow the human operator to translate and dilate the formation, but not to rotate it.[1]

In order to achieve the two aforementioned objectives we split the control inputs in two terms

$$(\boldsymbol{u}_i, \omega_i) = (\boldsymbol{\mu}_i^f, \omega_i^f) + (\boldsymbol{\mu}_i^m, 0). \tag{3}$$

The first term $(\boldsymbol{\mu}_i^f, \omega_i^f)$ is used to reach the desired bearing-formation and will be described in Sec. IV-A. The second term $(\boldsymbol{\mu}_i^m, 0)$, detailed in in Sec. IV-B, allows the human operator to control the translation and expansion of the formation without altering the bearing-formation.

III. BEARING-FORMATIONS

In our context a formation is a vector of configurations $\boldsymbol{q} = (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N) \in (\mathbb{R}^3 \times S^1)^N$. A bearing-formation is specified by constraining to certain fixed values all the relative bearings between the agents, i.e., a bearing-formation is the equivalence class of all the formations *realizing* the same relative bearings. We denote with $\mathcal{N}$ the set $\{(i,j) \in \{1, \ldots, N\}^2 | i \neq j\}$. A set of $N(N-1)$ relative bearings $\{{}^i\boldsymbol{\beta}_{ij} \in \mathbb{S}^2\}_{(i,j)\in\mathcal{N}}$ is *feasible* if there exists a formation realizing them (henceforth called a realization). Checking whether a set of bearings is feasible or not means verifying whether a system with $N(N-1)$ vectorial equations of the form of Eq. (2) in $4N$ scalar unknowns has at least a solution. Notice that if $N < 3$ any two relative bearings are feasible. In the following we assume a group of at least 3 agents.

In this Section we address three basic points about bearing-formations: (1) we define the remaining degrees of freedom of a formation (Prop. 1), (2) we find a minimal set of relative bearings sufficient to constrain all the other relative bearings, with cardinality linear in the number of robots (Prop. 2), and (3) we illustrate *synchronized motions*, i.e., the collective motions that keep constant all the relative bearings (Prop. 3). For the reader's convenience, all the proofs are in App. B.

Propositon 1: (Remaining degrees of freedom with constrained bearings) If a feasible set of bearings has a realization where the positions of at least three agents are not aligned, then all its realizations are defined up to a translation in $\mathbb{R}^3$, a rotation around the z axis, and a scale factor.

Propositon 2 (Minimal set of relative bearings): Given a feasible set of bearings $\{{}^i\boldsymbol{\beta}_{ij}\}_{(i,j)\in\mathcal{N}}$, if $\exists\ i,j$ s.t. $\left({}^i\boldsymbol{\beta}_{ik} \,\middle|\, -{}^iR_j{}^j\boldsymbol{\beta}_{jk}\right) \in \mathbb{R}^{3\times 2}$ is full-rank for every $k \neq i,j$ (i.e., agents i and j are not aligned with any other agent), then the relative bearings ${}^i\boldsymbol{\beta}_{ij}, {}^j\boldsymbol{\beta}_{ji}$ and ${}^i\boldsymbol{\beta}_{ik}, {}^j\boldsymbol{\beta}_{jk}, {}^k\boldsymbol{\beta}_{ki}$ for every $k \neq i,j$ are necessary and sufficient to uniquely constrain all the remaining bearings of the formation.

Considering together Prop. 1 and 2, we can conclude that if 2 agents are not aligned with any other agent, a set of $3N-4$ bearings is sufficient to specify the whole formation up to a translation, scaling, and rotation.[2] Therefore, although the total number of bearings in a formation is quadratic in the number of agents N, the bearings can be specified by only constraining a suitable subset whose cardinality is linear in N.

For any i,j,k with $i \neq j$ we define $\hat{\boldsymbol{p}}_{ij} = \frac{\boldsymbol{p}_{ij}}{\delta_{ij}}$, $\gamma_{ijk} = \frac{\delta_{ik}}{\delta_{ij}}$, and $M = \begin{bmatrix} M' & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix}$, with $M' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The following result is instrumental for Prop. 3.

Lemma 1 (Bearing-invariant motions for 3 agents): Given 3 non-aligned agents 1, 2 and 3, the motions that keep the relative bearings constant are a sum the following three primitives:

1) $(\dot{\boldsymbol{p}}_i,\ \dot{\theta}_i) = (\boldsymbol{v},\ 0)$ (*synchronized translation*)
2) $(\dot{\boldsymbol{p}}_i,\ \dot{\theta}_i) = (-\omega M \boldsymbol{p}_{1i},\ \omega)$ (*syn. rotation around ag.* 1)
3) $(\dot{\boldsymbol{p}}_i,\ \dot{\theta}_i) = (\lambda\gamma_{12i}\hat{\boldsymbol{p}}_{1i},\ 0)$ (*syn. dilation around ag.* 1)

where $i = 1,2,3$, $\boldsymbol{v}$ is any velocity vector in $\mathbb{R}^3$, ω and λ are any two scalars in $\mathbb{R}$.

Note that knowledge of relative-bearings (and not of inter-distances) is sufficient to perform the synch. dilation in 3). In fact, $\gamma_{iji} = 0$, $\gamma_{ijj} = 1$, and, using cross-products, we have, $\forall k \neq i,j\ \gamma_{ijk} = \frac{\|{}^j\boldsymbol{\beta}_{ji} \times {}^j\boldsymbol{\beta}_{jk}\|}{\|{}^k\boldsymbol{\beta}_{ki} \times {}^k\boldsymbol{\beta}_{kj}\|}$. On the other hand, knowledge of inter-distances is needed to perform the rotation in 2).

In the following Prop. 3 we characterize the motions keeping the same bearing-formation for any number of agents N.

Propositon 3 (Bearing-invariant movements for N agents): Given a bearing-formation where two agents, w.l.o.g. 1 and 2, are not aligned with any other agent, the motions that keep the relative bearings constant are given by the linear combination $(\dot{\boldsymbol{p}}_h, \dot{\theta}_h) = ((v_x\, v_y\, v_z)^T, 0) + \omega(-M\boldsymbol{p}_{1h}, -1)$ $+\lambda(\gamma_{12h}\hat{\boldsymbol{p}}_{1h}, 0)$, for any $h = 1, \ldots, N$, $v_x, v_y, v_z, \omega, \lambda \in \mathbb{R}$, where we assumed $\hat{\boldsymbol{p}}_{hh} = \mathbf{0}$ by convention.

The terms v_x, v_y, v_z represent a uniform translation in any direction, ω a synchronized rotation around the vertical axis passing through the agent 1, and λ an isotropic dilation/contraction centered on the agent 1. By properly combining rotation (dilation) with translation we can achieve a rotation around any vertical axis, and a dilation w.r.t any point. For instance, setting $\lambda = 0$ and $(v_x\, v_y\, v_z)^T = M\boldsymbol{p}_{13}$ generates a rotation around the 3-rd agent.

IV. CONTROL OF THE SLAVE SIDE

A. Control of the Bearing-Formation

The slave-side controller realizes the desired bearing-formation by solving the following control problem:

Problem 1 (Bearing-formation control): Given a set of feasible desired bearings $\{{}^i\boldsymbol{b}_{ij}\}_{(i,j)\in\mathcal{N}}$, find a control

[1] Our results could be easily adapted to the case where a depth estimator can be used to perform a synchronized rotation.

[2] For any other set of feasible bearings in which such a pair of indexes i,j does not exists, a larger number of bearings would be needed.

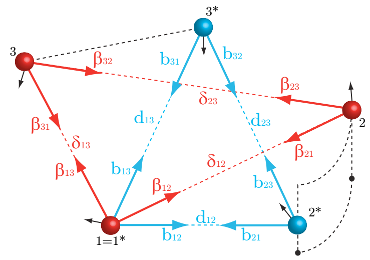

Fig. 1: Visual representation of the formation control law in the case of three agents. Agent 1 and 2 are the beacon agents. Agent 1 is stationary. Agent 2 rotates around 1 and moves vertically in order to reach the desired elevation. Agent 3 moves towards its desired scale-free position.

law $\{(\boldsymbol{\mu}_i^f, \omega_i^f)\}_{i=1,\ldots,N}$, depending only on the measured relative-bearings $\{{}^i\boldsymbol{\beta}_{ij}\}_{(i,j)\in\mathcal{N}}$, which steers ${}^i\boldsymbol{\beta}_{ij}$ to ${}^i\boldsymbol{b}_{ij}$, $\forall(i,j) \in \mathcal{N}$ and the distances $\{\delta_{ij}\}_{(i,j)\in\mathcal{N}}$ to a constant non-zero value.

We use the polar parametrization of a relative bearing ${}^i\boldsymbol{\beta}_{ij}$ in terms of elevation ${}^i\eta_{ij} \in [-\pi/2\ \pi/2]$ and azimuthal angle ${}^i\alpha_{ij} \in (-\pi\ \pi]$ defined by

$$ {}^i\boldsymbol{\beta}_{ij} = \left(\cos{}^i\eta_{ij}\cos{}^i\alpha_{ij}\ \ \cos{}^i\eta_{ij}\sin{}^i\alpha_{ij}\ \ \sin{}^i\eta_{ij}\right)^T, \quad (4) $$

denoting in brief ${}^i\boldsymbol{\beta}_{ij} \equiv ({}^i\alpha_{ij}, {}^i\eta_{ij})$. See App. A for a brush-up on polar coordinates. By convention we use greek symbols for measured quantities and the corresponding latin symbols for their desired values.

Propositon 4: Given a starting configuration described by the bearings $\{{}^i\boldsymbol{\beta}^0_{ij} \equiv ({}^i\alpha^0_{ij}, {}^i\eta^0_{ij})\}_{(i,j)\in\mathcal{N}}$ such that $\left\|{}^1\boldsymbol{\beta}^0_{1i} \times {}^2\boldsymbol{\beta}^0_{2i}\right\| \neq 0$, $\cos{}^1\eta^0_{12} \neq 0$, and a set of feasible desired bearings $\{{}^i\boldsymbol{b}_{ij} \equiv ({}^ia_{ij}, {}^ie_{ij})\}_{(i,j)\in\mathcal{N}}$ such that $\left\|{}^1\boldsymbol{b}_{1i} \times {}^2\boldsymbol{b}_{2i}\right\| \neq 0$, $\cos{}^1e_{12} \neq 0$ for all $i = 3,\ldots,N$, control law (5–9) asymptotically, and almost globally, steers ${}^i\boldsymbol{\beta}_{ij} \to {}^i\boldsymbol{b}_{ij}$ and $\delta_{ij} \to \bar{d}_{1i}\delta^0_{12}\cos{}^1\eta^0_{12}$, for any $(i,j) \in \mathcal{N}$

$$ \boldsymbol{\mu}_1^f = \mathbf{0} \qquad \omega_1^f = 0 \quad (5) $$

$$ \boldsymbol{\mu}_2^f = -\frac{K_p}{\cos{}^1\eta_{12}}\Big[\sin({}^1\alpha_{12} - {}^1a_{12})M\,{}^1\boldsymbol{\beta}_{12} + \left(\left(\sec{}^1\eta_{12}{}^1\boldsymbol{\beta}_{12} - \sec{}^1e_{12}{}^1\boldsymbol{b}_{12}\right)\cdot\hat{\boldsymbol{z}}\right)\hat{\boldsymbol{z}}\Big] \quad (6) $$

$$ \omega_2^f = K_\omega \sin({}^2\alpha_{21} - {}^2a_{21}) \quad (7) $$

$$ \boldsymbol{\mu}_i^f = -K_p\,{}^iR_1\left(\bar{\delta}_{1i}{}^1\boldsymbol{\beta}_{1i} - \bar{d}_{1i}{}^1\boldsymbol{b}_{1i}\right) \quad (8) $$

$$ \omega_i^f = \begin{cases} K_\omega\sin({}^i\alpha_{i1} - {}^ia_{i1}) & \text{if } \cos{}^ie_{i1} \neq 0 \\ K_\omega\sin({}^i\alpha_{i2} - {}^ia_{i2}) & \text{otherwise} \end{cases} \quad (9) $$

where $i = 3,\ldots,N$, $\hat{\boldsymbol{z}} = (0\,0\,1)^T$, $\bar{\delta}_{1i} = \gamma_{12i}\sec({}^1\eta_{12})$, δ^0_{12} is the initial inter-distance between agents 1 and 2, $\bar{d}_{1i} = \frac{\|{}^2\boldsymbol{b}_{21}\times{}^2\boldsymbol{b}_{2i}\|}{\|{}^i\boldsymbol{b}_{i1}\times{}^i\boldsymbol{b}_{i2}\|}\sec({}^1e_{12}) = \frac{d_{1i}}{d_{12}}\sec({}^1e_{12})$, ${}^i\boldsymbol{b}_{ij} \equiv ({}^ia_{ij}, {}^ie_{ij})$, K_p, K_ω are positive gains, and ${}^iR_1$ can be computed as $R^T(\alpha_{i1})R(\pi)R(\alpha_{1i})$, denoting by $R(*)$ the rotation matrix of a given angle around $\hat{\boldsymbol{z}}$.

Remark 1: Because of their special role, agents 1 and 2 will be denoted in the following as *beacon-agents*.

Control (5-6) keeps the position of agent 1 fixed. Hence, as it will be clear in Sec. IV-B, agent 1 is a suitable reference agent for the steering of the whole formation, since its translational dynamics will be affected only by the human commands (see Fig. 1). In addition, agent 2 only changes its altitude and rotates around agent 1 while keeping constant the length of the projection of the line between agent 1 and 2 (namely $\delta_{12}\cos{}^1\eta_{12}$). Notice also that the role of the beacon-agents in Prop. 4 can be taken by any other pair satisfying the Proposition hypotheses.

Communication complexity of (5–9) is linear in N since only agent 1 (resp. 2) is required to send to each agent $i = 3,\ldots,N$ the relative bearings ${}^1\boldsymbol{\beta}_{1i}$, ${}^1\boldsymbol{\beta}_{12}$ (resp. ${}^2\boldsymbol{\beta}_{2i}$, ${}^2\boldsymbol{\beta}_{21}$). The total number of measurements needed is linear in N since every agent $i = 3,\ldots,N$ needs to measure only the relative bearings ${}^i\boldsymbol{\beta}_{i1}$ and ${}^i\boldsymbol{\beta}_{i2}$. The number of measures needed by every agent but 1 and 2 is constant w.r.t N. Since each agent computes its control term $(\boldsymbol{\mu}_i^f, \omega_i^f)$ by only using its 2 measurements and 4 additional measurements received from 1 and 2, we can conclude that (5–9) is almost decentralized[3].

Control (5–9) is singular in some special zero-measure cases, i.e., when the i-th agent is aligned with agents 1 and 2, and when it is placed above agent 1 or 2. The practical approach used in Sec. VI is to apply a suitable constant control in the neighborhood of such singular configurations in order to quickly overcome these critical points.

B. Control of the Overall Motion

In this Section we design the term $\boldsymbol{\mu}_i^m$ of control (3) in order to steer the overall motion of the formation without affecting the bearing-formation. In particular we want to solve the following control problem:

Problem 2 (Overall-motion control): Given two reference agents, w.l.o.g. 1 and 2 (the beacon-agents), a translation velocity vector $\boldsymbol{\nu}^t \in \mathbb{R}^3$, and an expansion speed $r \in \mathbb{R}$, find a control law $(\boldsymbol{\mu}_i^m, 0)$ $(i = 1,\ldots,N)$ depending only on the measured relative bearings $\{{}^i\boldsymbol{\beta}_{ij}\}_{(i,j)\in\mathcal{N}}$ which regulates $\dot{\boldsymbol{p}}_1$ to $\boldsymbol{\nu}^t$ and $\dot{\delta}_{12}$ to r while keeping all the relative bearings $\{{}^i\boldsymbol{\beta}_{ij}\}_{(i,j)\in\mathcal{N}}$ constant.

In order to solve Prob. 2 we use the following controller:

$$ \boldsymbol{\mu}_i^m = {}^iR\boldsymbol{\nu}^t - r\gamma_{12i}{}^i\boldsymbol{\beta}_{i1}. \quad (10) $$

Propositon 5: Control (10) solves Prob. 2.

The human operator can control the translation and the rate of expansion of the whole formation by manipulating $\boldsymbol{\nu}^t$ and r respectively. Note that if $(\boldsymbol{\mu}_i^f, \omega_i^f)$ and $(\boldsymbol{\mu}_i^m, 0)$ are solution of resp. Prob. 1 and Prob. 2, then also $(\boldsymbol{\mu}_i^m + \boldsymbol{\mu}_i^f, \omega_i^f)$ is a solution of Prob. 1. This simple fact holds since $(\boldsymbol{\mu}_i^m, 0)$ does not affect the relative bearings. Moreover, if the reference agent 1 is also chosen as first beacon agent, its translational dynamics will be only affected by the command $\boldsymbol{\nu}^t$, resulting in an easier maneuverability of the swarm.

If (10) is executed by only a subset of the agents (including 1 and 2), the human would be still able to steer the whole group while approximately maintaining the bearing-formation. In fact, as the operator commands remain bounded (because of the limited workspace of the haptic devices), the

[3] A formation controller is called *centralized* if all the measurements taken from the whole agents are needed to compute every single control term for each agent, while is said to be *decentralized* if the control term of an agent i needs only quantities relative to itself and its neighbors retrieved by local communication and/or perception.

bearing-formation control would keep the bearing-formation error bounded as well. This fact has been empirically proven in our simulations, and a rigorous characterization will be addressed in future works.

V. THE MASTER SIDE

We use a 3DOF and a 1DOF force feedback devices in order to control translation and expansion rates of the agent formation. The 3DOF haptic device is modeled as

$$M(\boldsymbol{x}_t)\ddot{\boldsymbol{x}}_t + C(\boldsymbol{x}_t, \dot{\boldsymbol{x}}_t)\dot{\boldsymbol{x}}_t = \boldsymbol{\tau}_t + \boldsymbol{f}_t \tag{11}$$

where $\boldsymbol{x}_t \in \mathbb{R}^3$ is the configuration, $M(\boldsymbol{x}_t) \in \mathbb{R}^{3\times3}$ is the positive-definite/symmetric inertia matrix, $C(\boldsymbol{x}_t, \dot{\boldsymbol{x}}_t) \in \mathbb{R}^{3\times3}$ is the Coriolis matrix, and $\boldsymbol{\tau}_t, \boldsymbol{f}_t \in \mathbb{R}^3$ are the control and human forces, respectively. The 1DOF device is modeled as

$$m\ddot{x}_r = \tau_r + f_r \tag{12}$$

where $x_r \in \mathbb{R}$ is the position, $m \in \mathbb{R}^+$ is the mass, and $\tau_r, f_r \in \mathbb{R}^3$ are the control and human forces, respectively.

After having chosen the beacon agents 1 and 2, the tele-control is implemented by setting in (10)

$$\boldsymbol{\nu}^t = \lambda_t \boldsymbol{x}_t, \quad r = \lambda_r x_r, \tag{13}$$

where $\lambda_t > 0$ and $\lambda_r > 0$ are used suitable scaling factors from $(\boldsymbol{x}_t, x_r)$ to the desired agent velocities. Therefore the velocity commanded by the master to the i-th agent, in its local frame, results in:

$$\boldsymbol{v}_i^m = \lambda_t {}^iR\boldsymbol{x}_t - \lambda_r x_r \gamma_{12i} {}^i\boldsymbol{\beta}_{i1}. \tag{14}$$

which can be computed by the i-th agent using only local measurements and the measurements from agents 1 and 2 by means of local communication.

The UAVs are assumed to track the i-th agent velocity with sufficient precision. However, during the transients, the UAV *actual velocity* $\dot{\boldsymbol{q}}_i^\star$ will not track exactly the agent velocity $\dot{\boldsymbol{q}}_i$. In order to implement the tele-sensing, we provide the operator with two haptic cues proportional to the translation-velocity and expansion-speed tracking errors respectively, defined as

$$\boldsymbol{e}_t = \boldsymbol{x}_t - \boldsymbol{z}_t(t) \qquad e_r = x_r - z_r(t) \tag{15}$$

$$\boldsymbol{z}_t = \frac{1}{\lambda_t N}\sum_{i=1}^{N}(\lambda_r x_r \gamma_{12i} R_i {}^i\boldsymbol{\beta}_{i1} + R_i {}^i\dot{\boldsymbol{q}}^\star) \tag{16}$$

$$z_r = \frac{1}{\gamma_{12i}\lambda_r N}\sum_{i=1}^{N} {}^i\dot{\boldsymbol{q}}^\star \cdot {}^i\boldsymbol{\beta}_{i1} \tag{17}$$

The i-th UAV sends to the master device its current velocity in body frame ${}^i\dot{\boldsymbol{q}}_i^\star[k] = {}^iR\dot{\boldsymbol{q}}_i^\star[k]$, where the symbol $[k]$ indicates that the signal is received, sampled and discretized over the master-slave communication channel. The master controller uses all the received velocities in order to compute $\boldsymbol{z}_t[k]$ and $z_r[k]$, and implements the teleoperation controls as

$$\boldsymbol{\tau}_t = -B_t\dot{\boldsymbol{x}}_t - K_t\boldsymbol{x}_t - K_t^\star(\boldsymbol{x}_t - \bar{\boldsymbol{z}}_t[k]) \tag{18}$$

$$\tau_r = -B_r\dot{x}_r - K_r x_r - K_r^\star(x_r - \bar{z}_r[k]) \tag{19}$$

where B_t, B_r are a positive definite damping matrix whose role is to stabilize the master devices, K_t, K_r are diagonal matrix with non-negative entries (possibly all zeros) whose role is to give to the user the perception of the distance from the zero-commanded velocity, and $\bar{\boldsymbol{z}}_t(k)$, $\bar{z}_r(k)$ are the passive set-position modulation (PSPM) versions of $\boldsymbol{z}_t(t)$ and $z_r(t)$ respectively. By following the framework proposed in [5], we exploit here the PSPM algorithm [20] to ensure master passivity [21] w.r.t. the pairs (power ports) $(\boldsymbol{\tau}_t, \boldsymbol{z}_t)$ and (τ_r, z_r) with the control (18–19). Indeed, the PSPM action can enforce a passive behavior on the master also in presence of delays and packet losses in the communication channel (see [20] for details). This is sufficient to guarantee a stable interaction with a passive environment such as the human side [22] and our kinematic system, and thus an overall stable teleoperation.

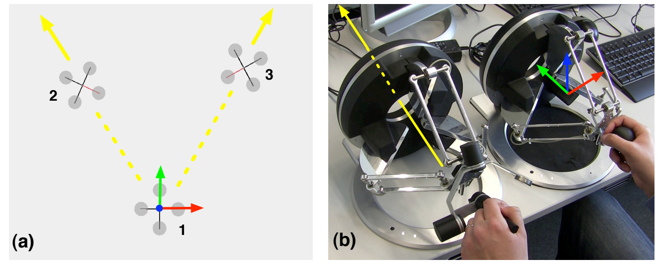

Fig. 2: Human/Hardware in-the-loop simulation setup in the case of 3 UAVs. Fig. 2-a: top-view of the physically simulated quadcopters. Fig. 2-b: two haptic devices used to get the motion commands and feed back the control forces.

VI. HUMAN/HARDWARE IN-THE-LOOP SIMULATIONS

We conducted several human/hardware in-the-loop simulations in order to prove the effectiveness of the proposed approach. We simulated a group quadcopters using the 3D engine OGRE as well as PhysX for the simulation of forces and physical interactions (see Fig, 2-a). Each UAV is controlled by a separate process which communicates via network with the simulator in the same way as it would communicate with a real robot. The translational command ν^t is provided by an Omega.3 haptic feedback device, that features 3 actuated degrees of freedom (DOF). An Omega.6 device, constrained to move in 1 DOF, is used to control the dilation rate r. The two devices are shown in Fig, 2-b The forces $\boldsymbol{\tau}_t$ in (18) and τ_r in (19) are presented to the human operator with a frequency of 2.5 kHz. The simulations are further documented in the accompanying video.

Here we present the result of a significant simulation where 10 UAVs start far away from the desired bearing-formation. The simulation is articulated in two phases: at first, the formation control alone is active, driving the UAVs to the desired bearing-formation; at the time $t = 24\,\mathrm{s}$ (vertical dashed black line in Fig. 3a-f), the human operator commands are enabled (translation, dilation) and the force feedback is activated on the haptic devices. While enabling the operator inputs from the very beginning would have not affected the performance of the formation control, this choice helps to highlight the effects of the different terms of the control. Fig. 3-e, shows the evolution of the average quadratic error of the bearing-formation, which goes exponentially

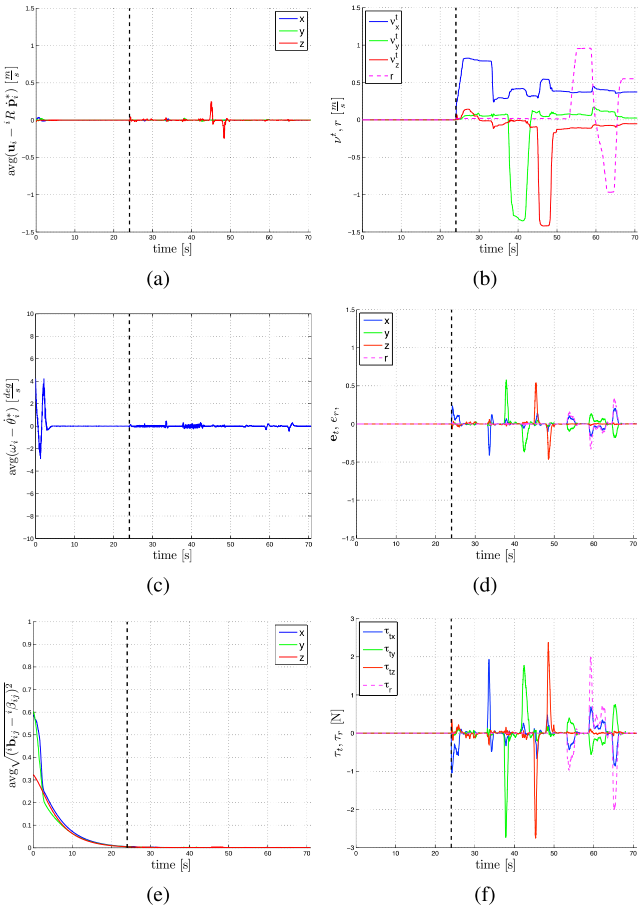

Fig. 3: Fig. 3-a,c: Average velocity and yaw rate tracking errors; Fig. 3-b: Operator commands: translation velocity and expansion-speed (dashed line); Fig. 3-d: Mismatch between commands and executions; Fig. 3-f: Force feedback; Fig. 3-e Mean square error w.r.t. desired bearing-formation.

to zero and does not increase when the human inputs are enabled. Fig. 3-a,c shows the average velocity and yaw rate tracking errors of the physically simulated UAVs w.r.t. the reference trajectory velocities of the agents in Eq. (1). As expected, due to the flatness of the quadrotor, the flight controller is able to keep the tracking error small. In Fig. 3-b the 3 components of the commanded translation velocity $\boldsymbol{\nu}^t$ and the commanded expansion-speed r) are depicted with solid and dashed lines respectively. For the sake of clarity, the simulation shows a sequence of all the elementary commands from Prop. 3, with a sequence of rapid translations along the main directions, followed by a dilation/contraction action. The mismatches between the commands and the actual translational/dilation velocities are represented in Fig. 3-d, and the force feedback provided to the operator in Fig. 3-e. The peaks of the mismatch, and consequently of the force feedback, correspond to high accelerations in the commands, and are due to the inertia of the physically simulated quadcopters. By means of this feedback the human operator is provided with a direct feeling of the remote-UAVs performances.

VII. CONCLUSIONS AND FUTURE WORK

In this paper we presented an innovative decentralized system for the bilateral teleoperation of groups of UAVs based on bearing-only measurements, i.e., without relying on distance measurements or global localization. A possible extension would be to (i) implement the proposed approach with real UAVs; (ii) estimate the inter-distances using the synchronized rotation, and (iii) use the estimated inter-distances in order to deal with the presence of obstacles.

VIII. ACKNOWLEDGMENTS

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APPENDIX

A. A Brush-up on Polar Coordinates

A bearing ${}^i\boldsymbol{\beta}_{ij} \in \mathbb{S}^2$ (unit sphere) can be parameterized in terms of the elevation angle ${}^i\eta_{ij} \in [-\pi/2\ \ \pi/2]$ and the azimuthal angle ${}^i\alpha_{ij} \in (-\pi\ \ \pi]$ as in (4). Note that ${}^i\boldsymbol{\beta}_{ij} = -{}^iR_j{}^j\boldsymbol{\beta}_{ji}$, ${}^2\eta_{21} = -{}^1\eta_{12}$, and ${}^iR_j = R^T({}^i\alpha_{ij})R(\pi)R({}^j\alpha_{ji})$, with $R(*)$ being the rotation matrix of a given angle along the z axis. This parametrization exhibits a singularity when $\cos{}^i\eta_{ij} = 0$, i.e., when agents i and j are placed on a vertical line, since ${}^i\alpha_{ij}$ is not defined.

In accordance with the polar coordinate convention, we define the unit vectors ${}^i\boldsymbol{\alpha}_{ij} = \frac{\partial{}^i\boldsymbol{\beta}_{ij}/\partial{}^i\alpha_{ij}}{\|\partial{}^i\boldsymbol{\beta}_{ij}/\partial{}^i\alpha_{ij}\|} = \left(-\sin{}^i\alpha_{ij}\ \ \cos{}^i\alpha_{ij}\ \ 0\right)^T = \frac{(\hat{\boldsymbol{z}}\times{}^i\boldsymbol{\beta}_{ij})}{\|\hat{\boldsymbol{z}}\times{}^i\boldsymbol{\beta}_{ij}\|}$ and ${}^i\boldsymbol{\eta}_{ij} = \frac{\partial{}^i\boldsymbol{\beta}_{ij}/\partial{}^i\eta_{ij}}{\|\partial{}^i\boldsymbol{\beta}_{ij}/\partial{}^i\eta_{ij}\|} = \left(-\sin{}^i\eta_{ij}\cos{}^i\alpha_{ij}\ \ -\sin{}^i\eta_{ij}\sin{}^i\alpha_{ij}\ \ \cos{}^i\eta_{ij}\right)^T = {}^i\boldsymbol{\beta}_{ij}\times{}^i\boldsymbol{\alpha}_{ij}$ with $\hat{\boldsymbol{z}} = (0\ 0\ 1)^T$. The dynamic equations of the azimuthal and elevation angles in terms of the controls in (1) follow from equations from the definitions

$$ {}^i\dot{\alpha}_{ij} = -\omega_i + \frac{1}{\delta_{ij}}({}^j\boldsymbol{\alpha}_{ji}^T\boldsymbol{u}_j - {}^i\boldsymbol{\alpha}_{ij}^T\boldsymbol{u}_i) \tag{20} $$

$$ {}^i\dot{\eta}_{ij} = \frac{1}{\delta_{ij}}({}^j\boldsymbol{\eta}_{ji}^T\boldsymbol{u}_j - {}^i\boldsymbol{\eta}_{ij}^T\boldsymbol{u}_i) \tag{21} $$

After some algebra, it is possible to find the dynamic equations for ${}^i\boldsymbol{\beta}_{ij}$ and δ_{ij}, where $i, j = 1, \ldots, N$, $i \neq j$

$$ \dot{\delta}_{ij} = -{}^j\boldsymbol{\beta}_{ji}^T\boldsymbol{u}_j - {}^i\boldsymbol{\beta}_{ij}^T\boldsymbol{u}_i \tag{22} $$

$$ \begin{aligned} {}^i\dot{\boldsymbol{\beta}}_{ij} = &-\omega_i{}^i\boldsymbol{\alpha}_{ij} + \frac{1}{\delta_{ij}}\left[({}^j\boldsymbol{\alpha}_{ji}^T\boldsymbol{u}_j - {}^i\boldsymbol{\alpha}_{ij}^T\boldsymbol{u}_i){}^i\boldsymbol{\alpha}_{ij} + \right.\\ &\left.({}^j\boldsymbol{\eta}_{ji}^T\boldsymbol{u}_j - {}^i\boldsymbol{\eta}_{ij}^T\boldsymbol{u}_i){}^i\boldsymbol{\eta}_{ij}\right]. \end{aligned} \tag{23} $$

B. Proofs

Proof: [of Prop. 1] To demonstrate the thesis we prove that, if the distance δ_{ij} is known for a pair i, j then the coordinates of the realization in the body frame of the agent i is unique.

Given a realization of the bearing formation, a translation in $\mathbb{R}^3$ of this realization is a new realization. In fact, such a translation does not modify neither the orientation of the agents nor their relative positions and thus the relative bearings defined as in (2) remain unaltered.

Any relative orientation ${}^{i}R_j$ can be computed directly from the equation ${}^{i}R_j{}^{j}\boldsymbol{\beta}_{ji} = -{}^{i}\boldsymbol{\beta}_{ij}$, except if ${}^{j}\boldsymbol{\beta}_{ji}$ (and hence also ${}^{i}\boldsymbol{\beta}_{ij}$) is parallel to $\hat{\boldsymbol{z}}$. In this last case the composition of more than one rotations with a third additional agent not aligned with i and j is needed. In short, as three agents are not aligned, it is possible to compute all their relative orientations and in particular the relative orientations w.r.t. i. Therefore the relative yaw angles between the agents expressed in the body frame of i are fixed, and a rotation around $\hat{\boldsymbol{z}}$ of the whole formation does not change them, thus yielding a new realization.

Finally, since we know the distance δ_{ij} between i and j, every other distance δ_{ki}, δ_{kj} can be computed by solving the triangulation equation $\left({}^{i}\boldsymbol{\beta}_{ik} \left| -{}^{i}R_j{}^{j}\boldsymbol{\beta}_{jk}\right.\right)(\delta_{ik}\ \ \delta_{jk})^T = \delta_{ij}{}^{i}\boldsymbol{\beta}_{ij}$ which has a unique solution *iff* the agent k is not aligned with i and j, i.e., iff $\left({}^{i}\boldsymbol{\beta}_{ik} \left| -{}^{i}R_j{}^{j}\boldsymbol{\beta}_{jk}\right.\right)$ is full-rank. The existence of such an agent k is guaranteed by the presence of three non-aligned agents. Scaling up/down δ_{ij} and every other distance δ_{ki}, δ_{kj} accordingly, does not modify the bearings in the triangulation thus resulting in a new realization. ∎

Proof: [of Prop. 2] The sufficiency is a direct consequence of the proof of Prop. 1. The minimality comes from the fact that the elimination of any bearing constraint increases the number of degrees of freedom. In fact each bearing constraint gives 2 independent equations (elevation and azimuth), but opposite bearings give only 3 independent equations, since the elevations are opposite in sign. Therefore the number of independent equations implied by the bearing constraints are: 3 for ${}^{i}\boldsymbol{\beta}_{ij}, {}^{j}\boldsymbol{\beta}_{ji}$; $3(N-2)$ for ${}^{i}\boldsymbol{\beta}_{ik}, {}^{k}\boldsymbol{\beta}_{ki}$ $(k \neq i, j)$; and only $(N-2)$ for ${}^{j}\boldsymbol{\beta}_{jk}$ $(k \neq i, j)$ since the elevation of ${}^{j}\boldsymbol{\beta}_{jk}$ must sum to 0 with the elevations of ${}^{j}\boldsymbol{\beta}_{jk}$ and ${}^{i}\boldsymbol{\beta}_{ij}$. The total number of equation is $4N-5$, which is exactly the number of constraints needed to have 5 degrees of freedom for a team of N agents. ∎

Proof: [of Lemma 1] From Prop. 2, it is sufficient to show that ${}^{1}\boldsymbol{\beta}_{12}$, ${}^{1}\boldsymbol{\beta}_{13}$, ${}^{2}\boldsymbol{\beta}_{21}$, ${}^{2}\boldsymbol{\beta}_{23}$ and ${}^{3}\boldsymbol{\beta}_{31}$ are maintained constant to prove that a motion does not change the bearing-formation. The time derivative of a bearing vector is obtained by differentiating (2), i.e., for ${}^{1}\boldsymbol{\beta}_{12}$ it is $R_1{}^{1}\dot{\boldsymbol{\beta}}_{12} = \omega_i M\hat{\boldsymbol{p}}_{12} + \frac{1}{\delta_{12}}P(\hat{\boldsymbol{p}}_{12})\dot{\boldsymbol{p}}_{12}$, where $P(\hat{\boldsymbol{p}}_{ij}) = (I - \hat{\boldsymbol{p}}_{ij}\hat{\boldsymbol{p}}_{ij}^T)$ with $i, j = 1, 2, 3$ and $i \neq j$ is the matrix projecting a vector in $\mathbb{R}^3$ on the plane perpendicular to $\hat{\boldsymbol{p}}_{ij}$. Then, by imposing ${}^{1}\dot{\boldsymbol{\beta}}_{12} = 0$ it results $0 = \omega_1 M\boldsymbol{p}_{12} + P(\hat{\boldsymbol{p}}_{12})\dot{\boldsymbol{p}}_{12}$. Similarly, from the other bearings follow the constraints

$$0 = \omega_2 M\boldsymbol{p}_{21} + P(\hat{\boldsymbol{p}}_{21})\dot{\boldsymbol{p}}_{21} \quad (24)$$
$$0 = \omega_1 M\boldsymbol{p}_{13} + P(\hat{\boldsymbol{p}}_{13})\dot{\boldsymbol{p}}_{13} \quad (25)$$
$$0 = \omega_3 M\boldsymbol{p}_{31} + P(\hat{\boldsymbol{p}}_{31})\dot{\boldsymbol{p}}_{31} \quad (26)$$
$$0 = \omega_2 M\boldsymbol{p}_{23} + P(\hat{\boldsymbol{p}}_{23})\dot{\boldsymbol{p}}_{23}, \quad (27)$$

where it has been exploited the fact that $\boldsymbol{p}_{ij} = -\boldsymbol{p}_{ji}$, $\dot{\boldsymbol{p}}_{ij} = -\dot{\boldsymbol{p}}_{ji}$ and $\hat{\boldsymbol{p}}_{ij} = -\hat{\boldsymbol{p}}_{ji}$ for $i, j = 1, 2, 3$ and $i \neq j$.

Consider now only the constraints relative to ${}^{1}\boldsymbol{\beta}_{12}$ and ${}^{2}\boldsymbol{\beta}_{21}$. By subtracting the second from the first it results $(\omega_1 - \omega_2)\, M\boldsymbol{p}_{12} = \omega_{12} M\boldsymbol{p}_{12} = 0$. This constraint is satisfied if either if $\hat{\boldsymbol{p}}_{12} = \hat{\boldsymbol{z}}$, which implies $M\boldsymbol{p}_{12} = 0$, or $\omega_{12} = 0$. Hereafter, we will assume that $\hat{\boldsymbol{p}}_{12} \neq \hat{\boldsymbol{z}}$, being this a more general and less restrictive situation. Therefore, constraint $\omega_{12} = 0$ will be used instead of (24). This same reasoning can be applied to replace (26) with $\omega_{13} = 0$.

The constraints (24–27) and the ones on ω_{12} and ω_{13} can be written in matrix form as

$$\underbrace{\begin{bmatrix} 0_{3\times3} & P(\hat{\boldsymbol{p}}_{12}) & 0_{3\times3} & M\boldsymbol{p}_{12} & 0 & 0 \\ 0_{3\times3} & 0_{3\times3} & P(\hat{\boldsymbol{p}}_{13}) & M\boldsymbol{p}_{13} & 0 & 0 \\ 0_{3\times3} & -P(\hat{\boldsymbol{p}}_{23}) & P(\hat{\boldsymbol{p}}_{23}) & M\boldsymbol{p}_{23} & 0 & 0 \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 1 & 0 \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 1 \end{bmatrix}}_{A_{11\times12}} \underbrace{\begin{bmatrix} \dot{\boldsymbol{p}}_1 \\ \dot{\boldsymbol{p}}_{12} \\ \dot{\boldsymbol{p}}_{13} \\ \omega_1 \\ \omega_{12} \\ \omega_{13} \end{bmatrix}}_{t_{12\times1}} = 0,$$

where it was also exploited the fact that $\dot{\boldsymbol{p}}_{23} = \dot{\boldsymbol{p}}_{13} - \dot{\boldsymbol{p}}_{12}$. Finally, the motions t that do not change the bearings of the formation are in the kernel of $A \in \mathbb{R}^{11\times12}$.

Consider now the first macro-row of the matrix A. The matrix $P(\hat{\boldsymbol{p}}_{12})$ spans the plane perpendicular to $\hat{\boldsymbol{p}}_{12}$, so its dimension is 2. The vector $M\boldsymbol{p}_{12}$ is perpendicular to $\hat{\boldsymbol{p}}_{12}$ owing to the structure of M, i.e., $P(\hat{\boldsymbol{p}}_{12})\, M\boldsymbol{p}_{12} = M\boldsymbol{p}_{12}$. Hence, the first macro-row of A has rank 2. With the same reasoning it can be shown that the second macro-row of A has rank 2 as well. The third macro-row has at least a row which is linearly independent w.r.t. the first macro row (since $-P(\hat{\boldsymbol{p}}_{23})$ is linearly independent w.r.t. $P(\hat{\boldsymbol{p}}_{12})$. In the end, considering the two final row we can conclude that it is rank$(A) \geq 7$. Lastly, the proof is concluded by the fact that the following 5 vectors, belonging to ker(A), are independent. First the vector $t_1 = \left(\dot{\boldsymbol{p}}^T\ 0_{1\times3}\ 0_{1\times3}\ 0\ 0\ 0\right)^T \in \mathbb{R}^{12\times3}$. In fact $At_1 = 0$. Synchronized translations are described by t_1. Second the vector $t_2 = \left(0_{1\times3}\ -\omega(M\boldsymbol{p}_{12})^T\ \omega(M\boldsymbol{p}_{13})^T\ -\omega\ 0\ 0\right)^T \in \mathbb{R}^{12}$. It is $At_2 = -\omega P(\hat{\boldsymbol{p}}_{12})M\boldsymbol{p}_{12} + \omega M\boldsymbol{p}_{12} - \omega P(\hat{\boldsymbol{p}}_{13})M\boldsymbol{p}_{13} + \omega M\boldsymbol{p}_{13} - \omega P(\hat{\boldsymbol{p}}_{23})M(\boldsymbol{p}_{13} - \boldsymbol{p}_{12}) + \omega M(\boldsymbol{p}_{13} - \boldsymbol{p}_{12}) = 0$ where it was exploited the property $P(\hat{\boldsymbol{p}}_{12})M\boldsymbol{p}_{12} = M\boldsymbol{p}_{12}$. Synchronized rotations around 1 are described by t_2. In the end the vector $t_3 = \left(0_{1\times3}\ \lambda\hat{\boldsymbol{p}}_{12}^T\ \lambda\gamma_{123}\hat{\boldsymbol{p}}_{13}^T\ 0\ 0\ 0\right)^T \in \mathbb{R}^{12}$. It is $At_3 = \lambda P(\hat{\boldsymbol{p}}_{12})\hat{\boldsymbol{p}}_{12} + \lambda\gamma_{123}P(\hat{\boldsymbol{p}}_{13})\hat{\boldsymbol{p}}_{13} + \frac{\lambda}{\delta_{12}}P(\hat{\boldsymbol{p}}_{23})(\boldsymbol{p}_{13} - \boldsymbol{p}_{12}) = 0$. since $P(\mathbf{v})\mathbf{v} = 0$. Synchronized dilations around 1 are described by t_3. ∎

Proof: [of Prop. 3] Assume w.l.o.g. that i, j are two agents not aligned with any other. By applying the same motion primitive from Lemma 1 to all the triplets of agents i, j and k, with $k = 1, \ldots, N$, $k \neq i, j$, the relative bearings ${}^{i}\boldsymbol{\beta}_{ij}$, ${}^{j}\boldsymbol{\beta}_{ji}$, ${}^{i}\boldsymbol{\beta}_{ik}$, ${}^{k}\boldsymbol{\beta}_{ki}$ and ${}^{j}\boldsymbol{\beta}_{jk}$ for each triplet are kept constant. Proposition 2 then guarantees that all the remaining bearings do not change. ∎

Proof: [of Prop. 4] From Prop. 2, it is sufficient to prove the convergence of ${}^{1}\boldsymbol{\beta}_{12}$, ${}^{2}\boldsymbol{\beta}_{21}$, ${}^{1}\boldsymbol{\beta}_{1i}$, ${}^{2}\boldsymbol{\beta}_{2i}$ and ${}^{i}\boldsymbol{\beta}_{i1}$ to prove the convergence of all the bearings.

Consider first the dynamics of ${}^{1}\alpha_{12}$. By injecting the controls (5-7) in (20) it results ${}^{1}\dot{\alpha}_{12} = -\dfrac{K_P}{\delta_{12}}\sin({}^{1}\alpha_{12} - {}^{1}a_{12})$ with $\delta_{12} > 0$, which proves that ${}^{1}\alpha_{12} \to {}^{1}a_{12}$ apart from the zero-measure condition ${}^{1}\alpha_{12} - {}^{1}a_{12} = \pm\pi$ which represents an unstable equilibrium for the closed-loop system. The dynamics of ${}^2\alpha_{21}$ can also be determined by applying the controls (5-7) to (20) as ${}^2\dot{\alpha}_{21} = -K_\omega \sin({}^2\alpha_{21} - {}^2a_{21}) - \frac{1}{\delta_{12}}\, {}^2\boldsymbol{\alpha}_{21}^T\, \boldsymbol{\mu}_2^f$, with $K_\omega > 0$. Since ${}^1\alpha_{12} \to {}^1a_{12}$, then $\boldsymbol{\mu}_2^f \to 0$ and therefore ${}^2\alpha_{21} \to {}^2a_{21}$. Finally, we can analyze the dynamics of ${}^1\eta_{12}$. By injecting the controls (5-6) in (21) it results ${}^1\dot{\eta}_{12} = -\frac{K_p}{\delta_{12}\cos{}^1\eta_{12}\cos{}^1e_{12}}\sin({}^1\eta_{12} - {}^1e_{12})$, with $\cos{}^1\eta_{12}, \cos{}^1e_{12} > 0$, hence ${}^1\eta_{12} \to {}^1e_{12}$. It follows also that ${}^2\eta_{21} = -{}^1\eta_{12} \to -{}^1e_{12} = {}^2e_{21}$. This proves that ${}^1\boldsymbol{\beta}_{12} \to {}^1\boldsymbol{b}_{12}$ and ${}^2\boldsymbol{\beta}_{21} \to {}^2\boldsymbol{b}_{21}$.

Before proving the convergence of the other bearings, we will show that the controls (5–6) do not change the product $\delta_{12}\cos{}^1\eta_{12}$. From the expressions of $\dot{\eta}_{ij}$ (21) and of $\dot{\eta}_{ij}$ (22) it follows that $\frac{\mathrm{d}\delta_{12}\cos{}^1\eta_{12}}{\mathrm{d}t} = -\sin{}^1\eta_{12}\left({}^2\boldsymbol{\eta}_{21}^T\boldsymbol{u}_2 - {}^1\boldsymbol{\eta}_{12}^T\boldsymbol{u}_1\right) - \cos{}^1\eta_{12}\left({}^2\boldsymbol{\beta}_{21}^T\boldsymbol{u}_2 + {}^1\boldsymbol{\beta}_{12}^T\boldsymbol{u}_1\right)$. Together with the controls (5,6) this yields that $\frac{\mathrm{d}\delta_{12}\cos{}^1\eta_{12}}{\mathrm{d}t} = (-\cos{}^2\alpha_{21} \;\; -\sin{}^2\alpha_{21} \;\; 0)\underbrace{(-A\sin{}^2\alpha_{21} \;\; A\cos{}^2\alpha_{21} \;\; *)^T}_{\boldsymbol{\mu}_2^f} = 0$, hence $\delta_{12}(t)\cos{}^1\eta_{12}(t) = \delta_{12}^0\cos{}^1\eta_{12}^0$ and $\delta_{12} \to \frac{\delta_{12}^0\cos{}^1\eta_{12}^0}{\cos{}^1e_{12}} = d_{12}$.

We consider now the convergence of ${}^1\boldsymbol{\beta}_{1i}$. Injecting the control (8) in the kinematic model of the agent ith and expressing P_i in the body frame of the 1st agent, it results ${}^1\dot{p}_i = -K_P\left(\frac{\delta_{1i}}{\delta_{12}\cos{}^1\eta_{12}}{}^1\boldsymbol{\beta}_{1i} - \bar{d}_{1i}{}^1\boldsymbol{b}_{1i}\right)$. Since $\delta_{12}(t)\cos{}^1\eta_{12}(t) = \delta_{12}^0\cos{}^1\eta_{12}^0$, the controller steers the position of the ith agent, ${}^1p_{1i} = \delta_{1i}{}^1\boldsymbol{\beta}_{1i}$, so that ${}^1\boldsymbol{\beta}_{1i} \to {}^1\boldsymbol{b}_{1i}$ and $\delta_{1i} \to \bar{d}_{1i}\delta_{12}^0\cos{}^1\eta_{12}^0 = d_{1i}$.

Consider now the triangulation $\delta_{2i}{}^2\boldsymbol{\beta}_{2i} = \delta_{12}{}^2\boldsymbol{\beta}_{21} + {}^2R_1\delta_{1i}{}^1\boldsymbol{\beta}_{1i}$. Having proven that ${}^2\boldsymbol{\beta}_{21} \to {}^2\boldsymbol{b}_{21}$, $\delta_{21} \to d_{12}$, ${}^1\boldsymbol{\beta}_{1i} \to {}^1\boldsymbol{b}_{1i}$ and $\delta_{1i} \to d_{1i}$, it is straightforward to prove that ${}^2\boldsymbol{\beta}_{2i} \to {}^2\boldsymbol{b}_{2i}$ and $\delta_{2i} \to \bar{d}_{2i}\delta_{12}^0\cos\eta_{12}^0 = d_{2i}$. The control law (9) acts on the dynamics of ${}^i\alpha_{i1}$ (see Eq. (20)) as ${}^i\dot{\alpha}_{i1} = -\frac{1}{\delta_{i1}}{}^i\boldsymbol{\alpha}_{i1}^T\boldsymbol{\mu}_i - K_\omega\sin({}^i\alpha_{i1} - {}^ia_{i1})$ and $\boldsymbol{\mu}_i \to \boldsymbol{0}$. Hence, we can conclude that ${}^i\boldsymbol{\beta}_{i1} \to {}^i\boldsymbol{b}_{i1}$. Finally, from Prop. 2 it follows that ${}^i\boldsymbol{\beta}_{ij} \to {}^i\boldsymbol{b}_{ij}$ and $\delta_{ij} \to \bar{d}_{1i}\delta_{12}^0\cos{}^1\eta_{12}^0$, for any $(i,j) \in \mathcal{N}$. ■

Proof: [of Prop. 5] The command (10) is a composition of a translation $\boldsymbol{\nu}^t$ and a dilation $r\gamma_{12i}\hat{\boldsymbol{p}}_{1i} = -r\gamma_{12i}\hat{\boldsymbol{p}}_{i1}$ expressed in world frame. From Prop. 3, it directly follows that this command does not change the relative bearings of the formation. Consider now the dynamics of δ_{hj}, obtained by applying (10) to (22): $\dot{\delta}_{hj} = -{}^j\boldsymbol{\beta}_{jh}^T({}^jR_h\boldsymbol{\nu}^t - r{}^j\boldsymbol{\beta}_{jh}) - {}^h\boldsymbol{\beta}_{hj}^T\boldsymbol{\nu}^t = {}^j\boldsymbol{\beta}_{jh}^T{}^j\boldsymbol{\beta}_{jh}r - ({}^j\boldsymbol{\beta}_{jh}^T{}^jR_h + {}^h\boldsymbol{\beta}_{hj}^T)\boldsymbol{\nu}^t = r$, where it was exploited the fact that ${}^j\boldsymbol{\beta}_{jh}^T{}^j\boldsymbol{\beta}_{jh} = I$ and ${}^j\boldsymbol{\beta}_{jh}^T{}^jR_h = {}^hR_j{}^j\boldsymbol{\beta}_{jh} = {}^h\boldsymbol{\beta}_{jh} = -{}^h\boldsymbol{\beta}_{hj}$. This proves the thesis. ■

References

[1] D. Lee and M. W. Spong, "Bilateral teleoperation of multiple cooperative robots over delayed communication network: theory," in *2005 IEEE Int. Conf. on Robotics and Automation*, Barcelona, Spain, Apr. 2005, pp. 360–365.

[2] D. Lee, "Semi-autonomous teleoperation of multiple wheeled mobile robots over the internet," in *2008 ASME Dynamic Systems and Control Conference*, Ann Arbor, MI, Oct. 2008.

[3] E. J. Rodríguez-Seda, J. J. Troy, C. A. Erignac, P. Murray, D. M. Stipanović, and M. W. Spong, "Bilateral teleoperation of multiple mobile agents: Coordinated motion and collision avoidance," *IEEE Trans. on Control Systems Technology*, vol. 18, no. 4, pp. 984–992, 2010.

[4] A. Franchi, P. Robuffo Giordano, C. Secchi, H. I. Son, and H. H. Bülthoff, "A passivity-based decentralized approach for the bilateral teleoperation of a group of UAVs with switching topology," in *2011 IEEE Int. Conf. on Robotics and Automation*, Shanghai, China, May 2011, pp. 898–905.

[5] D. Lee, A. Franchi, P. Robuffo Giordano, H. I. Son, and H. H. Bülthoff, "Haptic teleoperation of multiple unmanned aerial vehicles over the internet," in *2011 IEEE Int. Conf. on Robotics and Automation*, Shanghai, China, May 2011, pp. 1341–1347.

[6] A. Howard, L. E. Parker, and G. S. Sukhatme, "Experiments with a large heterogeneous mobile robot team: Exploration, mapping, deployment and detection," *International Journal of Robotics Research*, vol. 25, no. 5-6, pp. 431–447, 2006.

[7] M. Schwager, B. J. Julian, and D. Rus, "Optimal coverage for multiple hovering robots with downward facing cameras," in *2009 IEEE Int. Conf. on Robotics and Automation*, Kobe, Japan, May 2009, pp. 3515–3522.

[8] J. Fink, N. Michael, S. Kim, and V. Kumar, "Planning and control for cooperative manipulation and transportation with aerial robots," *International Journal of Robotics Research*, vol. 30, no. 3, 2010.

[9] B. Hannaford, "Stability and performance tradeoffs in bi-lateral telemanipulation," in *1989 IEEE Int. Conf. on Robotics and Automation*, Scottsdale, AZ, May 1989, pp. 1764–1767.

[10] A. K. Das, R. Fierro, V. Kumar, P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Trans. on Robotics and Automation*, vol. 18, no. 5, pp. 813–825, 2002.

[11] E. N. Johnson, A. J. Calise, R. Sattigeri, Y. Watanabe, and V. Madyastha, "Approaches to vision-based formation control," in *43th IEEE Conf. on Decision and Control*, Jan. 2004, pp. 1643–1648.

[12] O. A. A. Orqueda and R. Fierro, "Robust vision-based nonlinear formation control," in *2006 American Control Conference*, Minneapolis, MN, Jun. 2006, pp. 1422–1427.

[13] N. Moshtagh, N. Michael, A. Jadbabaie, and K. Daniilidis, "Vision-based, distributed control laws for motion coordination of nonholonomic robots," *IEEE Trans. on Robotics*, vol. 25, no. 4, pp. 851–860, 2009.

[14] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215 –233, 2007.

[15] M. Fliess, J. Lévine, P. Martin, and P. Rouchon, "Flatness and defect of nonlinear systems: Introductory theory and examples," *International Journal of Control*, vol. 61, no. 6, pp. 1327–1361, 1995.

[16] M. J. V. Nieuwstadt and R. M. Murray, "Real-time trajectory generation for differentially flat systems," *International Journal on Robust and Nonlinear Control*, vol. 8, p. 995–1020, 1998.

[17] V. Mistler, A. Benallegue, and N. K. M'Sirdi, "Exact linearization and noninteracting control of a 4 rotors helicopter via dynamic feedback," in *10th IEEE Int. Symp. on Robots and Human Interactive Communications*, Bordeaux, Paris, France, Sep. 2001, pp. 586–593.

[18] S. Bouabdallah and R. Siegwart, "Backstepping and sliding-mode techniques applied to an indoor micro," in *2005 IEEE Int. Conf. on Robotics and Automation*, May 2005, pp. 2247–2252.

[19] N. Guenard, T. Hamel, and R. Mahony, "A practical visual servo control for an unmanned aerial vehicle," *IEEE Trans. on Robotics*, vol. 24, no. 2, pp. 331–340, 2008.

[20] D. J. Lee and K. Huang, "Passive-set-position-modulation framework for interactive robotic systems," *IEEE Trans. on Robotics*, vol. 26, no. 2, pp. 354–369, 2010.

[21] C. Secchi, S. Stramigioli, and C. Fantuzzi, *Control of Interactive Robotic Interfaces: a port-Hamiltonian Approach*, ser. Springer Tracts in Advanced Robotics. Springer, 2007.

[22] N. Hogan, "Controlling impedance at the man/machine," in *1989 IEEE Int. Conf. on Robotics and Automation*, Scottsdale, AZ, May 1989, pp. 1626–1631.