# A Distributed Strategy for Generalized Biconnectivity Maintenance in Open Multi-robot Systems

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*Abstract*— **Preserving the connectivity of the underlying interaction graph in a multi-robot system is a necessary condition for allowing the group of robots to achieve a common task by resorting to only local information. However, in the context of *open* multi-robot systems, that is, when the number of robots in the team is not fixed, merely preserving connectivity of the current graph does not prevent the loss of connectivity after a robot joins/leaves the group. We present a distributed strategy to achieve *biconnectivity*, instead of simple connectivity, for a group of robots that allows establishment/deletion of interaction links as well as addition/removal of agents at anytime while guaranteeing that the connectivity, and thus functionality, of the team is always preserved. The proposed approach is completely distributed and embeds into a unique gradient-based control multiple constraints and requirements: (*i*) limited inter-robot communication ranges, (*ii*) limited field of view, (*iii*) desired inter-agent distances, and (*iv*) collision avoidance. Numerical simulations illustrate the effectiveness of our approach.**

## I. Introduction

Multi-robot systems where the number of robots in the team is not fixed, commonly referred to as Open Multi-Robot Systems (OMRS), are starting to become more relevant as applications shift from simple tasks to more complex and long-term collaborative missions where there is a need to account for real-world limitations such as limited battery charge, malfunction of the robots, or targeted attacks. In OMRS, and in multi-agent systems in general, it is well known that preserving the connectivity of the underlying interaction graph is a necessary condition for allowing the team to achieve a common task by resorting to only local information. This becomes even more evident when the interactions among the agents are determined by the existence of limited sensor-based relative measurements, such as distance or relative bearing.

Many approaches are proposed in the literature for connectivity maintenance in multi-robot systems with limited sensing, either by guaranteeing the maintenance of all the initially existing links in the team, e.g. [1]–[3], or by allowing the creating/deletion of links among the robots as long as the global connectivity of the group is preserved [4]–[6]. In a context of OMRS, however, merely preserving connectivity of the underlying graph is not enough to guarantee the existence of functionality of the team. Indeed, connectivity may be lost as robots join/leave the group due to, e.g., new mission specifications or failures, leading to the loss of functionality of the team. Therefore, it is necessary to design algorithms that not only maintain connectivity but that render the graph robust to node removals.

Although considerably less studied, some works have addressed fault-tolerant and robust control for multi-agent systems [7]–[9] or (local) connectivity maintenance for open multi-agent systems with limited communication range (albeit considering only the addition of agents) [10]. In particular, works such as [11]–[14] propose control algorithms to maintain the so-called *biconnectivity* of the graph [15], that is, the property of a graph to remain connected after one (or several) of the nodes and all its incident edges are removed. However, most of these only consider biconnectivity *maintenance* assuming that the initial graph is biconnected, which might not be the case when biconnectivity is lost after the addition/removal of an agent.

In this paper we propose a distributed gradient-based controller for achievement of (global) biconnectivity in the context of an OMRS, thereby guaranteeing the maintenance of connectivity, and thus of functionality, of the system after agents are added/removed. The contributions with respect to our previous work [14] and the related works in the literature mentioned above are: *(i)* we address this problem for second-order systems using the port-Hamiltonian formulation; *(ii)* we consider multiple inter-agent constraints and requirements embedded into a *generalized biconnectivity* measure, namely, limited inter-robot communication ranges, limited field of view, desired inter-agent distances, and collision avoidance; *(iii)* we show that the multi-robot system in closed-loop with the biconnectivity control law is passive, enabling the possibility to execute additional exogenous (passive) tasks besides the sole biconnectivity-maintenance action, increasing the versatility and applicability of our approach. In light of this, this paper can be seen as an extension of [4] to *distributed generalized biconnectivity* for *open multi-robot systems*.

This paper is organized as follows. In Section II are presented the robot and communication models. The generalized biconnectivity control is presented in Section III for multi-robot systems and adapted in Section IV to the context of OMRS. Finally, numerical simulations are presented in Section V and concluding remarks are given in Section VI.

## II. Preliminaries

### A. Robot model

Let $\mathcal{W} : \{\mathbf{O}_\mathcal{W}, \mathbf{X}_\mathcal{W}, \mathbf{Y}_\mathcal{W}, \mathbf{Z}_\mathcal{W}\}$ represent a world frame with $\mathbf{Z}_\mathcal{W}$ aligned with the vertical (gravity) direction. We

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consider the robots as floating point-mass agents with yaw orientation. Let us denote by $x_i \in \mathbb{R}^d$, $d \in \{2,3\}$ and $\psi_i \in S^1$, respectively, the position in $\mathcal{W}$ and the yaw angle about $\mathbf{Z}_{\mathcal{W}}$. Furthermore, let $v_i \in \mathbb{R}^d$ and $\omega_i \in \mathbb{R}$ be, respectively, the *body-frame* linear velocity and yaw rate. Define $\eta_i^\top = \begin{bmatrix} x_i^\top & \psi_i \end{bmatrix}$ and $\nu_i^\top = \begin{bmatrix} v_i^\top & \omega_i \end{bmatrix}$. Then, following the modeling assumptions of [5], [16] the kinematics of agent $i$ are

$$\dot{\eta}_i = J(\eta_i)\nu_i, \quad J(\eta_i) =: \begin{bmatrix} R_i & 0 \\ 0 & 1 \end{bmatrix}, \tag{1}$$

where $R_i = R_z(\psi_i) \in SO(3)$ is the rotation matrix associated with the yaw angle around $\mathbf{Z}_{\mathcal{W}}$. Now, letting $p_i = M_i \nu_i \in \mathbb{R}^{d+1}$ and $M_i \in \mathbb{R}^{(d+1)\times(d+1)}$ be, respectively, the generalized momentum and the generalized positive-definite inertia matrix, we use the port-Hamiltonian (pH) formalism to model the dynamics of agent $i$ as an element storing kinetic energy

$$\begin{cases} \dot{p}_i = F_i^\lambda + F_i^e - B_i M_i^{-1} p_i \\ \nu_i = \frac{\partial \mathcal{K}_i}{\partial p_i} = M_i^{-1} p_i \end{cases} \quad i = 1, \ldots, N(t), \tag{2}$$

where $\mathcal{K}_i(p_i) := \frac{1}{2} p_i^\top M_i^{-1} p_i$ is the kinetic energy stored by the agent during its motion, and $B_i \in \mathbb{R}^{(d+1)\times(d+1)}$ is a positive-definite matrix representing a velocity damping term. The force $F_i^\lambda \in \mathbb{R}^{(d+1)}$ represents the generalized biconnectivity force that will be designed below—see Section III-C—and $F_i^e \in \mathbb{R}^{(d+1)}$ is an additional input that can be exploited for implementing other tasks of interest. We refer the reader to [17] for an introduction to port-Hamiltonian modeling and control of robotic systems.

### B. Sensing and communication model

We assume that the robots are equipped with an omni-directional distance sensor and a camera-like sensor with limited field-of-view (FOV) used for detecting other robots in the group. In particular, we assume that when robot $j$ is in visibility of robot $i$, the latter can obtain from its onboard camera a measurement of the relative position of agent $j$ in its body-frame, i.e.,

$$x_{ij} = R_i^\top (x_i - x_j). \tag{3}$$

The distance sensor is in turn used for detecting other robots within a given sensing range.

The interaction between robots is modeled by a *time-varying* graph $\mathcal{G}(\mathcal{V}(t), \mathcal{E}(t))$ where the set of nodes $\mathcal{V}(t) := \{1, 2, \ldots, N(t)\}$ corresponds to the labels of the $N(t)$ agents with and the set of edges, $\mathcal{E}(t) \subseteq \mathcal{V}(t)^2$ represents the sensing/communication between a pair of nodes, that is, the presence of an edge $e_k := (i,j) \in \mathcal{E}(t)$ indicates that agent $j$ has access to information from node $i$ and vice-versa. Note that the time-dependence of the graph is due to the fact that both the edge set and the node set are time varying since in an open multi-agent system the number of agents is not fixed. We let $\mathcal{N}_i(t) = \{j \in \mathcal{V}(t) : (i,j) \in \mathcal{E}(t)\}$ be the set of neighbors of agent $i$. This interaction is modeled by the adjacency matrix $A \in \mathbb{R}^{N(t)\times N(t)}$, where an element $a_{ij} > 0$ if and only if $j \in \mathcal{N}_i(t)$, and $a_{ij} = 0$ otherwise. Due to the limited FOV of the robots, the interaction graph would generally be *directed* as visibility of robot $j$ by robot $i$ does not imply the inverse, i.e. $a_{ij} \neq a_{ji}$. However, in this work we consider that if a robot can sense another, the pair is able to communicate and therefore the graph is considered to be *undirected*. In Section III-C we discuss how we guarantee the symmetry of the weights, i.e., $a_{ij} = a_{ji}$.

The Laplacian matrix $L \in \mathbb{R}^{N(t)\times N(t)}$ is a symmetric positive semi-definite matrix given by $L = \text{diag}\{A\mathbf{1}\} - A$, with $\mathbf{1} \in \mathbb{R}^{N(t)}$ the vector of all ones. It is well known that some fundamental properties of the graph are associated with the Laplacian matrix. Specifically, denoting $\lambda_2$ as the second smallest eigenvalue of $L$, commonly referred to as the *algebraic connectivity*, we have $\lambda_2 > 0$ if and only if $\mathcal{G}$ is connected and $\lambda_2 = 0$ otherwise [18]. Furthermore, let $\nu_2$ denote the unit-norm eigenvector of $L$ associated with $\lambda_2$.

*Remark 1:* For clarity and ease of notation, in Section III we present first the generalized biconnectivity control design and passivity analysis considering a multi-robot system with a *fixed* number of agents and we drop the time dependence of the graph sets. Then, in Section IV we present the additional considerations needed to adapt our approach to an OMRS. •

## III. Generalized biconnectivity control

### A. Generalized connectivity

We first recall the definition of generalized connectivity introduced in [4]. As mentioned above, the adjacency matrix $A$ of a graph usually indicates the presence of an interaction link among a pair of agents $(i,j)$ by setting the corresponding elements $a_{ij} = a_{ji} = 1$, and $a_{ij} = a_{ji} = 0$ if no information can be exchanged at all. However, beyond existence of interaction, $A$ can be defined to also encode a number of additional inter-agent behaviors and constraints to be fulfilled by the group as a whole. This is achieved by designing the elements $a_{ij}$ so that the graph $\mathcal{G}$ decreases its degree of connectivity either when any two agents lose ability to physically exchange information under a given sensing model or any of the existing inter-agent behaviors or constraints is not met with the required accuracy even if the agents could still be able to interact from a pure sensing/communication standpoint.

More precisely, in this paper we define the weights $a_{ij}$ of the adjacency matrix as the product of four sub-weights

$$a_{ij} = \gamma_{ij} f_{ij} \alpha_{ij} \beta_{ij}. \tag{4}$$

The weight $\gamma_{ij} \geq 0$ encodes the maximum communication range, measuring the "quality" of the mutual information exchange, i.e. $\gamma_{ij} = 0$ if no exchange is possible and $\gamma_{ij} > 0$ otherwise. $f_{ij} \geq 0$ encodes FOV constraints, i.e. $f_{ij} > 0$ if agent $j$ is inside the FOV of agent $i$ and $f_{ij} = 0$ otherwise. The weight $\alpha_{ij} \geq 0$ represents other hard requirements that must be satisfied by agents, such as inter-agent collision avoidance, i.e. $\alpha_{ij} \to 0$ when the distance of agent $i$ to agent $j$ becomes smaller than some safety threshold. $\beta_{ij} \geq 0$ is meant to account for additional inter-agent soft requirements that should be preferably realized by the pair $(i,j)$ such as, e.g., formation control where $\beta_{ij}$ could have a unique maximum at some desired inter-agent distance and $\beta_{ij} \to 0$ as the distance deviates too much from the desired set-point.

By defining the weights $a_{ij}$ of the adjacency matrix in (4) as sufficiently smooth functions of the agents' relative positions and orientations, the (generalized) algebraic connectivity $\lambda_2$ becomes a smooth measure of the graph connectivity and, in particular, a smooth function of the system state. For instance, note that failure to comply with a hard constraint ($\gamma_{ij}$, $\alpha_{ij}$, $f_{ij}$) will then result in a null $i$th row (and $i$th column) in the adjacency matrix, necessarily leading to a disconnected graph, $\lambda_2 \to 0$. Therefore, we can design a gradient-like controller on the value of $\lambda_2$ to simultaneously guarantee connectivity maintenance as well as other hard and soft requirements on the multi-robot systems as has been previously done, e.g., in [4], [5], [19].

### B. Perturbed graph and perturbed algebraic connectivity

Although $\lambda_2$ is a (smooth) indicator of the generalized connectivity of the graph, in an OMRS context merely preserving connectivity is not enough to guarantee the maintenance of functionality of the team since it can be lost when a robot joins/leaves the group. Therefore, in order to consider the resilience of a graph with respect to a node addition/removal we design a control law to enforce (generalized) *biconnectivity*, which is the property of a graph to be resilient to a node removal [15]. More precisely, let $\mathcal{G}_{-i}$ be the graph remaining after the removal of node $i$. Then, $\mathcal{G}$ is said to be biconnected if, for any $i \in \mathcal{V}$, $\mathcal{G}_{-i}$ is connected. If $\mathcal{G}_{-i}$ is disconnected, node $i$ is called an *articulation* point. Therefore, an equivalent definition for a biconnected graph is for it to be a connected graph with no articulation points.

To characterize if a node is an articulation point we follow the approach in [14] in which each node checks its *locally biconnectivity*. Let $\mathcal{G}_i^l \subset \mathcal{G}$ denote the local subgraph centered at node $i$ and formed by the neighbors of node $i$, without itself. That is, $\mathcal{G}_i^l = (\mathcal{V}_i^l, \mathcal{E}_i^l)$, where $\mathcal{V}_i^l = \mathcal{N}_i$ and an edge $e_{kj} \in \mathcal{E}_i^l$ exits if and only if $e_{kj} \in \mathcal{E}$, with $k,\ j \in \mathcal{N}_i$. Then, a node is called locally biconnected if the second smallest eigenvalue $\lambda_{2,i}^l$ of the local graph $\mathcal{G}_i^l$ is positive. It is shown in [20] that a sufficient condition for biconnectivity of a graph is that all its nodes are locally biconnected. We associate each agent with a dynamic parameter $\rho_i \in [0, 1-\delta]$, with $\delta > 0$ a small constant, given by

$$\dot{\rho}_i = -\kappa_1 \rho_i + \frac{\kappa_2}{2}\left(1 + \text{sign}(\sigma_\lambda - \lambda_{2,i}^l)\right), \tag{5}$$

where we choose $\kappa_1$ and $\kappa_2$ such that $\frac{\kappa_2}{\kappa_1} = 1 - \delta$. The dynamic system (5) can be seen as an exponentially stable system with an additive disturbance which is different from zero only when $\lambda_{2,i}^l$ is smaller than a small threshold $\sigma_\lambda$. This means that if a node $i$ is not locally biconnected, i.e. $\lambda_{2,i}^l = 0$, the value of $\rho_i$ increases until it reaches the maximum value of $\frac{\kappa_2}{\kappa_1} = 1 - \delta$.

*Remark 2:* In an undirected graph, to characterize the local subgraph, each node only needs to receive the positions of its neighbors. Then, based on these, the local Laplacian matrices can be determined. •

Then, letting $\varepsilon_i = 1 - \rho_i$ we can define the *perturbed adjacency matrix* $\tilde{A}$, whose elements are given by

$$\tilde{a}_{ij} = \min\{\varepsilon_i, \varepsilon_j\} a_{ij}. \tag{6}$$

Note that when an $\varepsilon_i$ is small, i.e. $\varepsilon_i = \delta$, the perturbed adjacency matrix $\tilde{A}$ captures the case in which node $i$ is *almost* removed from the graph. Indeed, note that $\delta \to 0$ would correspond to the case where node $i$ is effectively removed. On the other hand, when $\varepsilon_i = 1$, for all $i \in \mathcal{V}$, the perturbed adjacency matrix $\tilde{A}$ coincides with the adjacency matrix $A$. Similarly, the perturbed Laplacian matrix is given by $\tilde{L} = \text{diag}\{\tilde{A}\mathbf{1}\} - \tilde{A}$. Let $\tilde{\lambda}_2$ be the second smallest eigenvalue of $\tilde{L}$ and $\tilde{\nu}_2$ its associated eigenvector. Then, the so called *perturbed algebraic connectivity* is defined as $\hat{\lambda}_2 = \lim_{\delta \to 0^+} \frac{\tilde{\lambda}_2}{\delta}$ and approximated by

$$\hat{\lambda}_2 \approx \tilde{\lambda}_2 / \delta. \tag{7}$$

The main idea behind (5) and (6) is to set $\varepsilon_i = \delta$ (equivalently $\rho_i = 1 - \delta$ ) if node $i$ is an articulation point to "perturb" the adjacency matrix and, hence, the algebraic connectivity of the graph. Then, akin to the generalized connectivity maintenance control [4], [5], based on the perturbed algebraic connectivity we can define a generalized *biconnectivity* maintenance control. Indeed, as it is shown in [21] in the case of a single perturbed node and extended in [13], [14] to the case of multiple perturbed nodes, there exists a lower bound $\bar{\lambda}$ such that if the perturbed algebraic connectivity satisfies $\hat{\lambda}_2 > \bar{\lambda}$ then the graph is biconnected. Therefore, we design a gradient-like law in order to maintain $\hat{\lambda}_2$ above a threshold $\bar{\lambda}$.

### C. Weights and control design

Before presenting the control design, we first define the four sub-weights in (4) encoding the multi-agent requirements and constraints.

Let us denote $d_{ij} := ||x_{ij}||$. The weight $\gamma_{ij} \geq 0$ encodes the maximum communication range, which is chosen to remain constant at a maximum value of 1 for $0 \leq d_{ij} \leq d_\gamma < D_\gamma$ and to smoothly vanish as $d_{ij} \to D_\gamma$, where $D_\gamma$ denotes the maximum communication range. More precisely we choose

$$\gamma_{ij}(d_{ij}) = \begin{cases} 1, & 0 \leq d_{ij} \leq d_\gamma \\ \frac{1}{2}\left(1 + \cos\left(\frac{\pi(d_{ij} - d_\gamma)}{D_\gamma - d_\gamma}\right)\right), & d_\gamma < d_{ij} \leq D_\gamma \\ 0, & D_\gamma < d_{ij}. \end{cases} \tag{8}$$

The soft constraints are encoded via the weight $\beta_{ij}(d_{ij})$ defined as a smooth function with a unique maximum at $d_{ij} = d_\beta$ and smoothly vanishing as $|d_{ij} - d_\beta| \to \infty$, i.e.,

$$\beta_{ij}(d_{ij}) = \exp\left(-\frac{(d_{ij} - d_\beta)^2}{\sigma}\right), \quad \sigma > 0. \tag{9}$$

Given a minimum safe distance $d_\alpha$ and a maximum range of influence among the agents $d_\alpha < D_\alpha \leq D_\gamma$ for collision avoidance, we first define a pairwise weight

$$\alpha^*_{ij}(d_{ij}) = \begin{cases} 0, & 0 \le d_{ij} \le d_\alpha \\ \frac{1}{2}\left(1 - \cos\left(\frac{\pi(d_{ij}-d_\alpha)}{D_\alpha - d_\alpha}\right)\right), & d_\alpha < d_{ij} \le D_\alpha \\ 1, & D_\alpha < d_{ij}, \end{cases} \tag{10}$$

which is constant at a maximum value of 1 for $D_\alpha < d_{ij}$ and smoothly vanishes as $d_{ij} \to d_\alpha$. Then, in order to force a disconnection of the graph when agent $i$ gets too close to *any* agent, we let $\mathcal{S}_i = \{j \in \mathcal{V} : \gamma_{ij} \neq 0\}$ be the sensing neighbors of agent $i$ and define the total weight as

$$\alpha_{ij}(d_{ij}) = \left(\prod_{k\in\mathcal{S}_i} \alpha^*_{ik}(d_{ik})\right)\left(\prod_{k\in\mathcal{S}_j\setminus\{i\}} \alpha^*_{jk}(d_{jk})\right), \tag{11}$$

where the second term guarantees $a_{ij} = a_{ji}$.

Finally, let $b_{ij} := x_{ij}/d_{ij}$ be the unit-norm bearing vector from $i$ to $j$ in the body-frame of robot $i$ and let $o_c$ be the fixed body-frame direction of the camera optical axis. Then, we let the cosine of the angle between $b_{ij}$ and $o_c$ be given by $c_{ij} = o_c^\top b_{ij}$ and we define a pairwise FOV constraint as

$$f_{ij}(c_{ij})^* = \begin{cases} 1, & 0 \le c_{ij} \le c_m \\ \frac{1}{2}\left(1 + \cos\left(\frac{\pi(c_{ij}-c_m)}{c_M - c_m}\right)\right), & c_m < c_{ij} \le c_M \\ 0, & c_M < c_{ij}, \end{cases} \tag{12}$$

where $c_M$ denotes the cosine of the maximum FOV angle and $c_m < c_M$. As mentioned in Section II-B above, unlike the weights $\gamma_{ij}$, $\beta_{ij}$, and $\alpha_{ij}$, the pairwise FOV constraints are asymmetric as visibility of robot $j$ by robot $i$ does not imply the inverse. Therefore, we proceed as in [5] and set the FOV weight as

$$f_{ij}(c_{ij}) = f_{ij}(c_{ij})^* + f_{ji}(c_{ji})^* - f_{ij}(c_{ij})^* f_{ji}(c_{ji})^*. \tag{13}$$

For the definition (13) we assume that if at least one of the robots in the pair $(i, j)$ looks at the other, a bidirectional communication link is established. In this way we guarantee that $f_{ij} = f_{ji}$ and avoid the overly constraining situation of needing robots $i$ and $j$ to simultaneously be inside each other's FOV.

Now we are able to present the design of the control law, which follows the connectivity preserving controller proposed in [22]. For this purpose let us define the set

$$\mathcal{D} := \{\hat\lambda_2 \in \mathbb{R}_{\ge 0} \mid \hat\lambda_2 > \bar\lambda\} \tag{14}$$

and introduce the scalar function $V_\lambda : \mathcal{D} \to \mathbb{R}_{\ge 0}$, $\hat\lambda_2 \mapsto V_\lambda(\hat\lambda_2)$, which is $\mathcal{C}^1$ over its domain and has the property that $V_\lambda(\hat\lambda_2) \to \infty$ as $\hat\lambda_2 \to \partial\mathcal{D}$, where $\partial\mathcal{D}$ denotes the border of $\mathcal{D}$. Then, the generalized biconnectivity force $F_i^\lambda$ in (2) is given by the gradient controller

$$F_i^\lambda = -\kappa \frac{\partial V_\lambda(\hat\lambda_2)}{\partial \eta_i} = -\kappa \frac{\partial V_\lambda}{\partial \hat\lambda_2} \begin{bmatrix} \frac{\partial \hat\lambda_2}{\partial x_i} \\ \frac{\partial \hat\lambda_2}{\partial \psi_i} \end{bmatrix}. \tag{15}$$

where, akin to [22] and recalling (7), we have that

$$\frac{\partial \hat\lambda_2}{\partial x_i} = \frac{1}{\delta} \sum_{j=1}^{N} R_i^\top \frac{\partial \tilde a_{ij}}{\partial x_i} (\hat\nu_i - \hat\nu_j)^2 \tag{16}$$

$$\frac{\partial \hat\lambda_2}{\partial \psi_i} = \frac{1}{\delta} \sum_{j=1}^{N} \frac{\partial \tilde a_{ij}}{\partial \psi_i} (\hat\nu_i - \hat\nu_j)^2. \tag{17}$$

Moreover choosing

$$V_\lambda(\hat\lambda_2) = \coth(\hat\lambda_2 - \lambda^*) - 1, \tag{18}$$

with $\lambda^* = \bar\lambda$ being the biconnectivity lower bound, we have

$$\frac{\partial V_\lambda}{\partial \hat\lambda_2} = \text{csch}^2(\hat\lambda_2 - \lambda^*). \tag{19}$$

The detailed expressions of the gradients in (16) and (17) can be found in the Appendix.

*Remark 3:* Although the formulation in (15) requires knowledge of the (perturbed) algebraic connectivity $\hat\lambda_2$, there exist numerous algorithms in the literature for estimating this global parameter in a distributed manner with a specified error bound—see e.g. [22]—even with a non-constant number of agents—see [23]. Therefore, we consider that the (perturbed) algebraic connectivity (or at least a good enough estimation) is known to the agents at all times. •

### D. Closed-loop stability

It is shown in [14] that for single-integrator systems under the gradient control (15), the set (14) is rendered forward invariant, or equivalently, biconnectivity maintenance is guaranteed for all $t$. In this paper we show that under the generalized biconnectivity force defined in (15), the multi-robot system is passive and maintains biconnectivity by rendering the set (14) forward invariant. For that purpose, inspired by [24], we define the *rotational incidence matrix* of the graph, but in a non-standard way, i.e., let

$$\begin{aligned} \mathcal{E}^* &= \{(1,2),(1,3),\dots,(1,N),\dots,(N-1,N)\} \\ &= \{e_1, e_2, \dots, e_{N-1}, \dots, e_{N(N-1)/2}\} \end{aligned} \tag{20}$$

be the set of all possible edges in $\mathcal{G}$, that is, all the pairs $(i, j)$ such that $i < j$, sorted in lexicographical order. Then, we define $E \in \mathbb{R}^{3N\times 3|\mathcal{E}^*|}$ such that, $\forall e_k = (i, j) \in \mathcal{E}^*$, $[E]_{ik} = R_i^\top$ and $[E]_{jk} = -R_j^\top$, if $e_k \in \mathcal{E}$, and $[E]_{ik} = [E]_{jk} = 0_{3\times 3}$ otherwise, where $R_i$ is the rotation matrix.

Furthermore, for analysis purposes, let us denote by $\tilde x_k = x_i - x_j$ the relative position expressed in the world frame, and by $\tilde\psi_k = \psi_i - \psi_j$ the relative orientation. Then, the relative pose is given by $\tilde\eta_k^\top = \begin{bmatrix}\tilde x_k^\top & \tilde\psi_k\end{bmatrix}$. Similarly we let $\tilde\nu_k = \nu_i - \nu_j$ be the relative *body-frame* velocities. Then, replicating the order used for $\mathcal{E}^*$ all the possible $|\mathcal{E}^*|$ relative poses and velocities are collected in the vectors $\tilde\eta^\top = \begin{bmatrix}\tilde\eta_1^\top \dots \tilde\eta_{N(N-1)/2}^\top\end{bmatrix}$ and $\tilde\nu^\top = \begin{bmatrix}\tilde\nu_1^\top \dots \tilde\nu_{N(N-1)/2}^\top\end{bmatrix}$. Moreover, note that, with $\partial \tilde x_k/\partial x_i = R_i^\top$, (15) may be expressed as

$$F_i^\lambda = -\sum_{k=1}^{3N(N-1)/2} \frac{\partial V^\lambda(\lambda_2)}{\partial \tilde\eta_k}. \tag{21}$$

Now, let the energy of the system be

$$H(p, \tilde\eta) = \sum_{i=1}^{N} \mathcal{K}_i(p_i) + V^\lambda(\lambda_2(\tilde\eta)) \ge 0. \tag{22}$$

Then, using the pH formulation, we have

$$\begin{bmatrix}\dot{p}\\ \dot{\tilde{\eta}}\end{bmatrix} = -\begin{bmatrix}B & -E\\ E^\top & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial p}\\ \frac{\partial H}{\partial \tilde{\eta}}\end{bmatrix} + \begin{bmatrix}F^e\\ 0\end{bmatrix} \tag{23}$$

$$\tilde{\nu} = \frac{\partial H}{\partial p}. \tag{24}$$

*Proposition 1:* Consider the closed-loop multi-robot system (23)-(24). If the initial graph $\mathcal{G}(t_0)$ is *biconnected*, then the system is passive with respect to the power port $(F^e, \tilde{\nu})$ and the set (14) is rendered forward invariant, i.e., biconnectivity is maintained for all $t \geq t_0$. □

*Proof:* The derivative of (22) along (23)-(24) satisfies

$$\begin{aligned}\dot{H}(p,\tilde{\eta}) =& -\frac{\partial^\top H}{\partial p} B \frac{\partial H}{\partial p} + \frac{\partial^\top H}{\partial p} F^e\\ \leq& \tilde{\nu}^\top F^e,\end{aligned} \tag{25}$$

implying that the multi-robot system is passive.

Furthermore, from (25) we conclude that $H$, therefore $V_\lambda$, is bounded along the trajectories of (23). In order to prove the forward invariance of $\mathcal{D}$ we proceed by contradiction. Let us assume that there exists a $\tau$ such that $\hat{\lambda}_2(t) \in \mathcal{D}$ for all $t \in [t_0, \tau)$ and $\hat{\lambda}_2(\tau) \notin \mathcal{D}$. Therefore, from continuity of the solutions we have that $\hat{\lambda}_2(t) \to \partial\mathcal{D}$ as $t \to \tau$. From the definition of $V_\lambda$ this implies that $V_\lambda(t, \hat{\lambda}_2(t)) \to \infty$. However, the latter is in contradiction with (25) which implies that $V_\lambda(t, \hat{\lambda}_2(t))$ is bounded. Therefore, we conclude that $\mathcal{D}$ is forward invariant implying that biconnectivity is maintained if the initial graph is biconnected. ■

## IV. Generalized biconnectivity for OMRS

In the previous section we presented a passivity-preserving control law for maintaining generalized biconnectivity on a multi-robot system that is initially biconnected. However, in an OMRS setting, where the agents can arbitrarily join/leave the system, biconnectivity may be lost after a node addition/removal, i.e., it may be possible that $\hat{\lambda}_2 \leq \bar{\lambda} = \lambda^*$. But, since $V_\lambda$ is only defined in the set (14), the generalized biconnectivity force (15) would not be able to render the graph biconnected, indeed it may not be well defined.

In order to overcome this problem in this section, we set instead $\lambda^*$ in (18) as a time-varying function $\lambda^* : \mathbb{R}_{\geq 0} \to [0, \bar{\lambda} + \epsilon]$,with $\epsilon > 0$ a small constant, that is reinitialized to zero when biconnectivity is lost and that smoothly increases to its maximum value $\bar{\lambda} + \epsilon$ to guarantee biconnectivity achievement. However, since $\lambda^*$ is a global parameter, we let each agent have a copy of the parameter, given as the solution to the dynamical system

$$\dot{\lambda}_i^* = f(\lambda_i^*) - \kappa_c \sum_{j \in \mathcal{N}_i} (\lambda_i^* - \lambda_j^*) - \kappa'_\lambda c_i(t) \lambda_i^* \tag{26}$$

where $\kappa_c, \kappa'_\lambda > 0$ and $f(\lambda_i^*) = -\kappa_\lambda(\lambda_i^* - (\bar{\lambda} + \epsilon))$ with $\bar{\lambda}$ the biconnectivity lower bound. $c_i(t) \in \{0, 1\}$ is a pinning variable that is used for re-initializing the value of $\lambda_i^*$ after biconnectivity is lost. More precisely, each agent has a (local) notion of biconnectivity based on the values of $\rho_i$ and $\lambda_{2,i}^l$ as explained in the previous section. Therefore, if an agent is (locally) biconnected (equivalently $\lambda_{2,i}^l > 0$) then $c_i(t) = 0$. On the other hand, if an agent senses that local biconnectivity is lost ($\dot{\rho}_i > 0$ or equivalently $\lambda_{2,i}^l \to 0$), it becomes "pinned", i.e. $c_i(t) = 1$. The underlying idea is that if the graph is biconnected, $c_i(t) = 0$ for all $i$ and, under (26), the values $\lambda_i^*(t)$ reach synchronization exponentially converging to $\bar{\lambda} + \epsilon$. Otherwise, if the graph losses biconnectivity then $c_i(t) = 1$ for at least one $i$, called the pinned node(s) which drive $\lambda_i^*(t) \to 0$, re-initializing their values to be able to recover biconnectivity.

Note that (26) is in the form of the systems studied in [25]. Therefore, for a sufficiently large $\kappa'_\lambda$, invoking [25, Theorem 1] the systems (26) reach synchronization. For now, let us denote $\tilde{\lambda}^*$ as the synchronized value of (26). Then, we can reformulate (18) as

$$V_\lambda(\hat{\lambda}_2) = \coth(\hat{\lambda}_2 - \tilde{\lambda}^*) - 1. \tag{27}$$

Then, we have that the derivative of (22) becomes

$$\begin{aligned}\dot{H}(p,\tilde{\eta}) =& -\frac{\partial^\top H}{\partial p} B \frac{\partial H}{\partial p} + \frac{\partial^\top H}{\partial p} F^e - \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \dot{\tilde{\lambda}}^*\\ \leq& \tilde{\nu}^\top F^e - \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \dot{\tilde{\lambda}}^*.\end{aligned} \tag{28}$$

When $\dot{\tilde{\lambda}}^* \geq 0$, from (28), passivity is preserved. This, mainly occurs when $c_i = 0$, for all $i$, $\lambda_i^*(t_0) = \lambda_j^*(t_0)$ for all $i \in \mathcal{V}$. However, note that it is possible that $\dot{\tilde{\lambda}}^* < 0$ when $c_i = 1$ for at least one $i$, that is, when biconnectivity is lost due to a node addition/removal. In such cases passivity is lost. Moreover, for implementation, instead of the synchronized valued $\tilde{\lambda}^*$, the local copy $\lambda_i^*$ is used in the biconnectivity force $F_i^\lambda$. Therefore, due to the synchronization error only an approximated version of the biconnectivity force, $\hat{F}_i^\lambda$, is actually applied. In order to deal with these problems, we rely on the concept of energy tanks [26].

Consider a tank with state $x_{ti} \in \mathbb{R}$ and its associated energy function $T_i = 0.5 x_{ti}^2 \geq 0$. Let $D_i = p_i^\top M_i^{-1} B_i M_i^{-1} p_i$ be the power dissipated by agent $i$ because of the damping. Then we set the augmented dynamics

$$\left\{\begin{array}{l} \dot{p}_i = F_i^e - w_i x_{ti} - B_i M_i^{-1} p_i\\ \dot{x}_{ti} = \frac{1}{x_{ti}} D_i + u_{ti} + w_i^\top \nu_i \qquad i = 1, \ldots, N(t).\\ \nu_i = \frac{\partial \mathcal{K}_i}{\partial p_i} = M_i^{-1} p_i \end{array}\right. \tag{29}$$

The main idea is to exploit the energy stored in the tank through the input $u_{ti}$ to render the system passive and to implement a desired force on agent $i$ through the input $w_i \in \mathbb{R}^{d+1}$. Here, we replace the biconnectivity force $F_i^\lambda$ by a passive implementation of its approximation $\hat{F}_i^\lambda$ setting

$$w_i = -\frac{1}{x_{ti}} \hat{F}_i^\lambda. \tag{30}$$

Furthermore, to account for the difference between $\hat{F}_i^\lambda$ and $F_i^\lambda$, we let

$$u_{ti} = \frac{\varsigma_i}{x_{ti}} \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \dot{\lambda}_i^*, \tag{31}$$

where $\varsigma_i = 1$ if $\dot{\lambda}_i^* < 0$ and $\varsigma_i = 0$ otherwise.

*Remark 4:* The dynamic extension of the system using the energy tanks is already implemented in [4] in the context of generalized connectivity maintenance in order to apply an *estimation* connectivity force using the *estimation* of the algebraic connectivity $\lambda_2$—cf. Remark 3. In this paper, the same can be done, in addition to the discrepancy on the biconnectivity force due to (26), by exploiting the energy-tank passive interconnection. We refer the readers to [4] for a more detailed explanation. •

Now, let the total system-plus-tanks energy be given by

$$H(p, \tilde{\eta}, x_t) = \sum_{i=1}^{N} \left(\mathcal{K}_i(p_i) + T_i(x_{ti})\right) + V^{\lambda}(\lambda_2(\tilde{\eta})) \geq 0. \tag{32}$$

Let also $\mathcal{W} := \text{diag}\{-w_i\} \in \mathbb{R}^{(d+1)N\times N}$, $\mathcal{P} := \text{diag}\{(1/x_{ti})p_i^\top M_i^{-1}\} \in \mathbb{R}^{N\times(d+1)N}$, $G_t^\top = [0\ 0\ I_N]$, $G^\top = [I_N \otimes I_3\ 0\ 0]$ with '$\otimes$' denoting the Kronecker product, and

$$\nabla H = \begin{bmatrix} \frac{\partial^\top H}{\partial p} & \frac{\partial^\top H}{\partial \tilde{\eta}} & \frac{\partial^\top H}{\partial x_t} \end{bmatrix}^\top.$$

The augmented closed-loop system in pH form becomes

$$\begin{bmatrix} \dot{p} \\ \dot{\tilde{\eta}} \\ \dot{x}_t \end{bmatrix} = \left[ \begin{pmatrix} 0 & E & \mathcal{W} \\ -E^\top & 0 & 0 \\ -\mathcal{W}^\top & 0 & 0 \end{pmatrix} - \begin{pmatrix} B & 0 & 0 \\ 0 & 0 & 0 \\ -\mathcal{P}B & 0 & 0 \end{pmatrix} \right] \nabla H \tag{33}$$

$$+ G_t u_t + G F^e$$

$$\tilde{\nu} = G^\top \nabla H \tag{34}$$

The derivative of (32) along (33)-(34) satisfies

$$\begin{aligned} \dot{H}(p, \tilde{\eta}, x_t) =& -\frac{\partial^\top H}{\partial p} B \frac{\partial H}{\partial p} + \frac{\partial^\top H}{\partial x_t} \mathcal{P} B \frac{\partial H}{\partial p} \\ &+ \nabla H^\top G F^e + \nabla H^\top G_t u_t - \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \dot{\bar{\lambda}}^* \\ \leq& \tilde{\nu}^\top F^e + \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \left( \sum_{i=1}^{N} \varsigma_i \dot{\lambda}_i^* - \dot{\bar{\lambda}}^* \right), \end{aligned} \tag{35}$$

where we used (30)-(31) and the definition of $\mathcal{P}$. Now, due to the fact that (26) reaches synchronization, we can assume that $|\dot{\bar{\lambda}}^* - (1/N)\sum_{i=1}^{N} \dot{\lambda}_i^*| \leq \epsilon_\lambda$. Then,

$$\dot{H}(p, \tilde{\eta}, x_t) \leq \tilde{\nu}^\top F^e + \epsilon_\lambda + \frac{1}{N} \frac{\partial V_\lambda}{\partial \hat{\lambda}_2} \sum_{i=1}^{N} \left( \varsigma_i N \dot{\lambda}_i^* - \dot{\lambda}_i^* \right), \tag{36}$$

and if $\dot{\lambda}_i^* \geq 0$, recalling (31), the second term on the right-hand side of (36) is negative. On the other hand, if $\dot{\lambda}_i^* < 0$, we have that the term inside the sum in the second term on the right-hand side of (36) yields $\sum_i (N-1)\dot{\lambda}_i^*$, which is also negative. Therefore, we have

$$\dot{H}(p, \tilde{\eta}, x_t) \leq \tilde{\nu}^\top F^e + \epsilon_\lambda. \tag{37}$$

Inequality (37) implies that the multi-robot system is (weakly) passive.

Now, we address the final considerations needed to adapt our approach to the context of OMRS. Due to the change of dimension of the state in an OMRS, produced by the addition/removal of agents, in order to properly analyze the system, the formalism of multi-mode multi-dimensional ($\text{M}^3$D) switched systems can be used, where the plant is modeled as a hybrid system switching between operational "modes," each with a different state dimension. This formalism has been previously used to study open multi-agent systems, e.g., in [10], [27]–[29]. In particular, in [29] a passivity-based framework is proposed to autonomously manage the addition/removal of agents while guaranteeing the preservation of passivity of the OMRS, using the pH and $\text{M}^3$D formalism and hybrid energy tanks. Due to the lack of space this complete $\text{M}^3$D is not included in this paper. However, note that the closed-loop system (33)-(34) together with the dynamic equation (26) can be considered as representing the evolution of the system between two switching instants, that is, when the number of agents is constant before and after adding/removing a robot. Then, since the system (between switching instants) is (weakly) passive from (35), by slightly adapting the energy tanks herein to have an impulsive switching behavior akin to the approach in [29], it can be shown that the OMRS is (weakly) passive with respect to the power port $(F^e(t), \tilde{\nu}(t))$.

## V. Numerical Examples

In this section we present some numerical examples to validate our theoretical results. We consider an OMRS with an initial number of agents $N(t_0) = 21$ starting from the initial configuration shown in Fig. 1. Note that the initial graph is connected but not biconnected. During the simulation time, some agents join the network at around 5, 55, and 65 seconds and others leave at 19.5 and 40 seconds, as can be seen from the evolution of $N(t)$ in Fig. 5. Figs. 2 and 3 show the positions of the agents and the graph at the switching instants $t = 19.5$ and $t = 40$ when agents leave. As can be seen from the latter, at these instants the remaining graph is still connected despite the agents that leave the network. Moreover in Fig. 4 is presented the final graph which is biconnected. In Fig. 6 is shown the evolution of the perturbed algebraic connectivity. As can be seen from Fig. 6 the perturbed algebraic connectivity remains at all times above the prescribed bound, which stabilizes (between switching instants) at the value $\bar{\lambda} = 0.5$, thereby guaranteeing that biconnectivity is recovered after each addition/removal and preserved afterwards.

## VI. Conclusions

We present a distributed approach to acquire and preserve the generalized global biconnectivity of the graph in a open multi-robot system where it it is fundamental to guarantee that the graph remains connected after an agent is added/removed. We consider that the interactions among the robots are sensor based and, besides the commonly considered limited inter-robot communication ranges for connectivity, we also encode into the biconnectivity measure

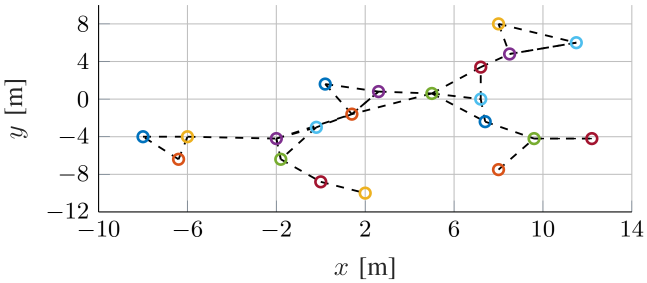


Fig. 1: Initial configuration and initial graph depicting also the field of view of the agents.

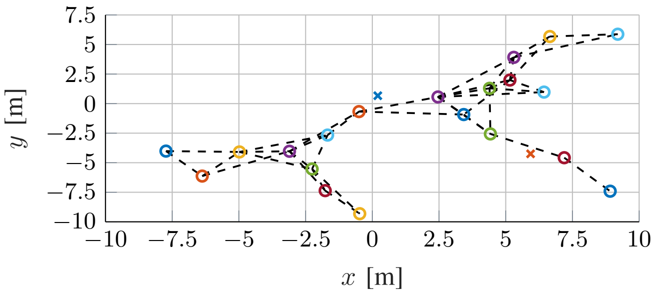


Fig. 2: Positions and graph at the switching time $t = 19.5$s. The agents that left are marked by the '$\times$'.

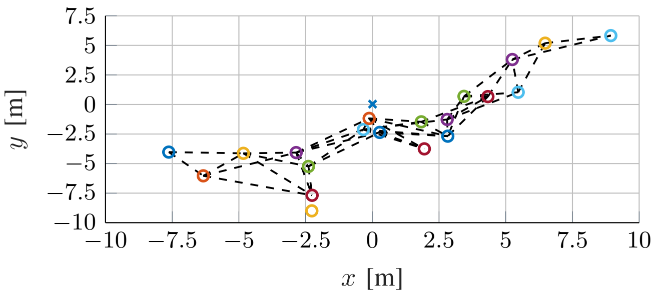


Fig. 3: Positions and graph at the switching time $t = 40$s. The agent that left is marked by the '$\times$'.

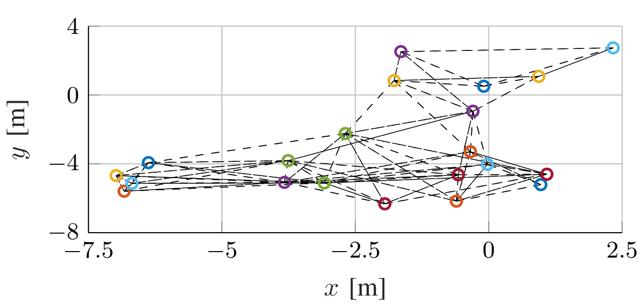


Fig. 4: Final positions and graph.

other constraints and requirements such as limited field of view, desired inter-agent distances, and collision avoidance. Furthermore, using the port-Hamiltonian representation we establish passivity of the system with respect to external inputs. Current and future work focus on considering a persistent shared control scenario of open multi-robot systems

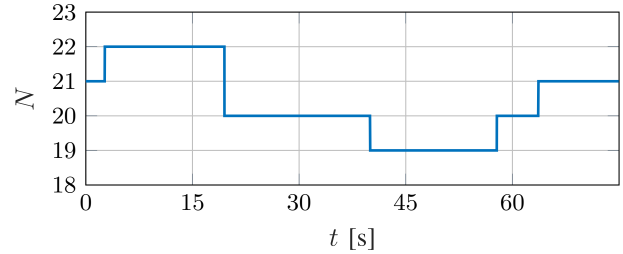


Fig. 5: Evolution of the number of agents in the network.

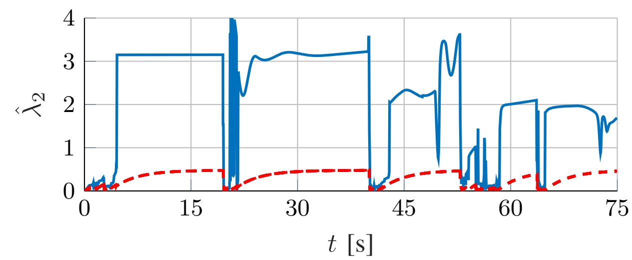


Fig. 6: Perturbed algebraic connectivity. The dashed lines represents the bounds $\lambda_i^*(t)$.

and implementing the results experimentally.

## Appendix

The detailed expressions of the weight gradient in (16)-(17) are given by

$$\frac{\partial \tilde{a}_{ij}}{\partial \varsigma_i} = \frac{\partial \gamma_{ij}}{\partial \varsigma_i} f_{ij}\alpha_{ij}\beta_{ij} + \gamma_{ij}\frac{\partial f_{ij}}{\partial \varsigma_i}\alpha_{ij}\beta_{ij} + \gamma_{ij} f_{ij}\frac{\partial \alpha_{ij}}{\partial \varsigma_i}\beta_{ij} + \gamma_{ij} f_{ij}\alpha_{ij}\frac{\partial \beta_{ij}}{\partial \varsigma_i} \tag{38}$$

with $\varsigma_i = \{x_i, \psi_i\}$. From (13), we have

$$\frac{\partial f_{ij}}{\partial x_i} = (1 - f_{ji}^*)\frac{\partial f_{ij}^*}{\partial \varsigma_i} + (1 - f_{ij}^*)\frac{\partial f_{ji}^*}{\partial \varsigma_i} \tag{39}$$

with $\varsigma_i = \{x_i, \psi_i\}$. Recalling (12) and that $c_{ij} = o_c^\top b_{ij}$, we obtain

$$\frac{\partial f_{ij}^*}{\partial x_i} = \frac{\partial f_{ij}^*}{\partial c_{ij}}\frac{R_i P_{ij}}{d_{ij}} o_c, \quad \frac{\partial f_{ji}^*}{\partial x_i} = \frac{\partial f_{ji}^*}{\partial c_{ji}}\frac{R_j P_{ji}}{d_{ij}} o_c, \tag{40}$$

where $P_{ij} := I_d - \beta_{ij}\beta_{ij}^\top$. Similarly, we get

$$\frac{\partial f_{ij}^*}{\partial \psi_i} = \frac{\partial f_{ij}^*}{\partial c_{ij}} o_c^\top S(b_{ij}), \tag{41}$$

and $\partial f_{ji}^*/\partial \psi_i = 0$, where $S(\cdot)$ is the skew-symmetric cross-product matrix.

On the other hand, since $\gamma_{ij}$, $\alpha_{ij}$, and $\beta_{ij}$ are only functions of the distance $d_{ij}$, it follows that

$$\frac{\partial s_{ij}}{\partial x_i} = \frac{\partial s_{ij}}{\partial d_{ij}}\frac{x_i - x_j}{d_{ij}}, \quad s \in \{\gamma, \alpha, \beta\}, \tag{42}$$

and $\partial s_{ij}/\partial \psi_i = 0$.

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