Quantifier elimination for robot positioning with landmarks of uncertain position

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We consider a mobile robot equipped with a camera that moves in a 2D environment. We aim to compute a domain for the pose (position and orientation) $\mathbf{r} = (x, y, \theta)$ in which we are sure the robot is situated [1, 3]. The robot knows landmarks positions $X_w(x_w, y_w)$ in the world frame, and measures their projection u in the image (represented in normalized coordinates, assuming known camera calibration parameters). The perspective projection of a world point in the camera frame is given as follows:

$$\begin{cases} u &= f(\mathbf{r}, X_w) = \frac{1}{z_c}\left((x_w - x)\cos(\theta) - (y_w - y)\sin(\theta)\right) \\ z_c &= (x_w - x)\sin(\theta) + (y_w - y)\cos(\theta) \; > 0 \end{cases} \tag{1}$$

Problem The problem amounts in determining the set of all poses that are consistent with 1-D image measurement and its 2-D world corresponding point which is known with a bounded error. Assuming the image measurement uncertainty is represented by the interval $[u]$ and the landmarks positions are known to be inside the box $[X_w]$, the pose domain is the solution of the following quantified set inversion [2] problem:

$$S = \{\mathbf{r} \in \mathbb{R}^2 \times [0, 2\pi],\, \exists X_w \in [X_w], f(\mathbf{r}, X_w) \in [u], z_c > 0\} \tag{2}$$

Since the expression of f in Eq. 1 is not well posed when z_c gets close to 0, the constraints $f(\mathbf{r}, X_w) \in [u], z_c > 0$ can be re-written as two inequalities:

$$g : \begin{cases} g_1 : (x_w - x)(\cos(\theta) - \underline{u}\,\sin(\theta)) - (y_w - y)(\sin(\theta) + \underline{u}\,\cos(\theta)) > 0 \\ g_2 : (x_w - x)(\cos(\theta) - \overline{u}\,\sin(\theta)) - (y_w - y)(\sin(\theta) + \overline{u}\,\cos(\theta)) < 0 \end{cases}$$

where $\underline{u}$ and $\overline{u}$ denote the lower bounds and the upper bound of $[u]$.

Quantified set inversion with a branch and bound algorithm s.a. SIVIA requires bisecting also on the quantified variables (otherwise, parts of the domain may remain undetermined). We therefore need to eliminate the quantifier ($\exists$) to avoid bisecting on the landmark position box $[X_w]$.

Quantifier free equations We adopt a geometric approach to perform the elimination. Let $\mathcal{C} = \{(\underline{x_w}, \underline{y_w}), (\underline{x_w}, \overline{y_w}), (\overline{x_w}, \overline{y_w}), (\overline{x_w}, \underline{y_w})\}$, the set of corners formed by the box $[X_w]$. The pose is found consistent if g_1 is verified by at least one corner from C, g_2 is also verified by at least one corner, and the box $[X_w]$ is not behind the camera.

$$S = \left\{ \mathbf{r} \in \mathbb{R}^2 \times [0, 2\pi], \bigvee_{X_w \in \mathcal{C}} g_1(\mathbf{r}, X_w, [u]), \bigvee_{X_w \in \mathcal{C}} g_2(\mathbf{r}, X_w, [u]), \neg \text{behind}(\mathbf{r}, X_w) \right\}$$

The first two conditions can also be written as:

$$\begin{cases} \max_{X_w \in \mathcal{C}} \left((x_w - x)(\cos(\theta) - \underline{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \underline{u} \cos(\theta)) \right) > 0 \\ \min_{X_w \in \mathcal{C}} \left((x_w - x)(\cos(\theta) - \overline{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \overline{u} \cos(\theta)) \right) < 0 \end{cases}$$

The $\neg$behind constraint expressing the fact that the camera is front-looking is given by:

$$\begin{cases} \cos\alpha \geq 0 \implies x < \overline{x_w} \\ \cos\alpha \leq 0 \implies x > \underline{x_w} \\ \sin\alpha \geq 0 \implies y < \overline{y_w} \\ \sin\alpha \leq 0 \implies y > \underline{y_w} \end{cases} \Leftrightarrow \begin{cases} \cos\theta - u\sin\theta < 0 \;\vee\; x < \overline{x_w} \\ \cos\theta - u\sin\theta > 0 \;\vee\; x > \underline{x_w} \\ \sin\theta + u\cos\theta > 0 \;\vee\; y > \underline{y_w} \\ \sin\theta + u\cos\theta < 0 \;\vee\; y < \overline{y_w} \end{cases}$$

with α the observed ray angle in the world frame:

Results An example with 3 uncertain landmarks is depicted in Fig. 1.

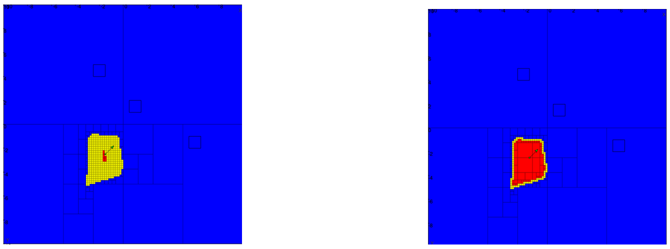

Figure 1: Pose domain in X-Y plane without and with quantifier elimination.

References

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