Addendum to ”*Visual Servoing through mirror reflection*”

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***Abstract*— This short note is an addendum to [2]. We prove that the interaction related to a point in the case of a mobile camera observing a point through a planar mirror reflection is given by (E 34)[1] and that this case is similar to the classical interaction matrix of a virtual point.**

In the case the camera is mounted on the end effector of the robot and observes the scene through its reflection in a planar mirror. As can be seen on Figure 1, this is a classical visual servoing configuration [1] with respect to ${}^c\mathbf{X}_R$. The corresponding interaction matrix $\mathbf{L_x}$ is thus classically defined by $\dot{\mathbf{x}} = \mathbf{L_x}{}^c\mathbf{v}_c$ where ${}^c\mathbf{v}_c = ({}^c\boldsymbol{v}_c, {}^c\boldsymbol{\omega}_c)^\top$ are the translational and rotational velocity of the camera expressed in the camera frame $\mathcal{F}_c$. $\mathbf{L_x}$ is then given by:

$$\mathbf{L_x} = \begin{pmatrix} -1/Z_R & 0 & x/Z_R & xy & -(1+x^2) & y \\ 0 & -1/Z_R & y/Z_R & 1+y^2 & -xy & -x \end{pmatrix} \quad (1)$$

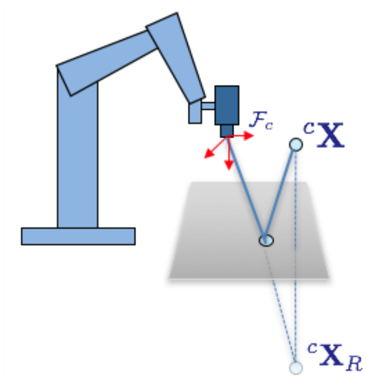

Fig. 1. Case of a controlled camera.

Proof. In that case, the camera is moving and the mirror is static. When we derive the reflection equation (E 5) it has to be noted that the motion of the camera induces a motion (in the camera frame $\mathcal{F}_c$) of the mirror and a motion of the point (note that in the case reported in Section IV [2] only a motion of the mirror had to be considered since the point was static wrt the camera). Therefore we have:

$$\begin{aligned} {}^c\dot{\mathbf{X}}_R = & \, 2\mathbf{n}\dot{d} + 2\dot{\mathbf{n}}d - 2(\dot{\mathbf{n}}\mathbf{n}^\top + \mathbf{n}\dot{\mathbf{n}}^\top){}^c\mathbf{X} \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)\, {}^c\dot{\mathbf{X}} \end{aligned} \quad (2)$$

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[1]Equation (E x) refers to equation (x) in [2].

From equation (E 14), (E 16), (2) can be rewritten as

$$\begin{aligned} {}^c\dot{\mathbf{X}}_R = & \, 2\mathbf{n}\dot{d} - 2\dot{\mathbf{n}}d + 2(\dot{\mathbf{n}}\mathbf{n}^\top - \mathbf{n}\dot{\mathbf{n}}^\top){}^c\mathbf{X}_R \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)\begin{pmatrix} -\mathbf{I}\,{}^c\boldsymbol{v}_c & [{}^c\mathbf{X}]_\times\,{}^c\boldsymbol{\omega}_c \end{pmatrix} \end{aligned} \quad (3)$$

If we separate the translational ${}^c\dot{\mathbf{X}}_{R|{}^c\boldsymbol{v}_c}$ and rotational ${}^c\dot{\mathbf{X}}_{R|{}^c\boldsymbol{\omega}_c}$ components of ${}^c\dot{\mathbf{X}}_R$, from (E 12) we have for the translational component:

$${}^c\dot{\mathbf{X}}_{R|{}^c\boldsymbol{v}_c} = -2\mathbf{n}\mathbf{n}^\top\,{}^c\boldsymbol{v}_c - (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top){}^c\boldsymbol{v}_c = -\mathbf{I}\,{}^c\boldsymbol{v}_c \quad (4)$$

leading to $\mathbf{L}_{\boldsymbol{v}} = -\mathbf{I}$. For the rotational component, alternatively to (E 21), we can write using (E 17):

$$\begin{aligned} (\dot{\mathbf{n}}\mathbf{n}^\top - & \,\mathbf{n}\dot{\mathbf{n}}^\top){}^c\mathbf{X}_R \\ & = -\mathbf{n}([\mathbf{n}]_\times{}^c\boldsymbol{\omega}_m)^\top{}^c\mathbf{X}_R + ([\mathbf{n}]_\times{}^c\boldsymbol{\omega}_m)\mathbf{n}^\top{}^c\mathbf{X}_R \\ & = \mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times{}^c\boldsymbol{\omega}_m + \mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times{}^c\boldsymbol{\omega}_m \end{aligned} \quad (5)$$

leading to

$$\begin{aligned} {}^c\dot{\mathbf{X}}_{R|{}^c\boldsymbol{\omega}_c} = & \,(-2d[\mathbf{n}]_\times + 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times + 2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[{}^c\mathbf{X}]_\times\,){}^c\boldsymbol{\omega}_c \end{aligned} \quad (6)$$

or

$$\begin{aligned} \mathbf{L}_{\boldsymbol{\omega}} = & - 2d[\mathbf{n}]_\times + 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times + 2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[{}^c\mathbf{X}]_\times \\ = & - 2d[\mathbf{n}]_\times + 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times + 2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[(\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)\,{}^c\mathbf{X}_R + 2d\mathbf{n}]_\times \\ = & -2d[\mathbf{n}]_\times + 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times + 2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[{}^c\mathbf{X}_R]_\times + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[-2\mathbf{n}\mathbf{n}^\top\,{}^c\mathbf{X}_R]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[2d\mathbf{n}]_\times \\ = & -2d[\mathbf{n}]_\times + 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times + 2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times \\ & + [{}^c\mathbf{X}_R]_\times - 2\mathbf{n}\mathbf{n}^\top[{}^c\mathbf{X}_R]_\times \\ & + (\mathbf{I} - 2\mathbf{n}\mathbf{n}^\top)[-2\mathbf{n}\mathbf{n}^\top\,{}^c\mathbf{X}_R]_\times \\ & + 2d[\mathbf{n}]_\times - 4d\mathbf{n}\mathbf{n}^\top[\mathbf{n}]_\times \\ = & \,2\mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times + [{}^c\mathbf{X}_R]_\times \\ & - 2[\mathbf{n}\mathbf{n}^\top\,{}^c\mathbf{X}_R]_\times + 4\mathbf{n}\mathbf{n}^\top[\mathbf{n}\mathbf{n}^\top\,{}^c\mathbf{X}_R]_\times \end{aligned} \quad (7)$$

$\mathbf{n}^\top{}^c\mathbf{X}_R$ is a scalar. This leads to, first, $[\mathbf{n}\mathbf{n}^\top\;{}^c\mathbf{X}_R]_\times = [\mathbf{n}]_\times\mathbf{n}^\top\;{}^c\mathbf{X}_R$ and thus to $\mathbf{n}\mathbf{n}^\top[\mathbf{n}\mathbf{n}^\top\;{}^c\mathbf{X}_R]_\times = \mathbf{n}\mathbf{n}^\top[\mathbf{n}]_\times\mathbf{n}^\top\;{}^c\mathbf{X}_R = 0$ since $\mathbf{n}^\top[\mathbf{n}]_\times = 0$; second $[\mathbf{n}\mathbf{n}^\top\;{}^c\mathbf{X}_R]_\times = \mathbf{n}^\top{}^c\mathbf{X}_R[\mathbf{n}]_\times$ which finally leads to:

$$\mathbf{L}_{\boldsymbol{\omega}} = [{}^c\mathbf{X}_R]_\times \quad (8)$$

We thus have

$${}^c\dot{\mathbf{X}}_R = \begin{pmatrix} -\mathbf{I} & [{}^c\mathbf{X}_R]_\times \end{pmatrix}\begin{pmatrix} {}^c\boldsymbol{v}_c \\ {}^c\boldsymbol{\omega}_c \end{pmatrix} \quad (9)$$

From equations (E 6) and (9), it is easy to obtain (1) [1]. □

References

[1] F. Chaumette and S. Hutchinson. Visual servoing and visual tracking. In B. Siciliano and O. Khatib, editors, *Handbook of Robotics*, chapter 24, pages 563–583. Springer, 2008.

[2] E. Marchand and F. Chaumette. Visual servoing through mirror reflection. In *IEEE Int. Conf. on Robotics and Automation, ICRA'17*, Singapore, May 2017.